US009459632B2

(12) United States Patent
Houston et al.

(10) Patent No.: US 9,459,632 B2
(45) Date of Patent: Oct. 4, 2016

(54) **SYNCHRONIZED ARRAY OF VIBRATION ACTUATORS IN A NETWORK TOPOLOGY**

(71) Applicant: **Coactive Drive Corporation**, New York, NY (US)

(72) Inventors: **John Houston**, New York, NY (US); **Rob Morris**, Cincinnati, OH (US); **Nathan Delson**, San Diego, CA (US); **Gabe Graham**, Beaverton, OR (US)

(73) Assignee: **Coactive Drive Corporation**, New York, NY (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 25 days.

(21) Appl. No.: **14/382,976**

(22) PCT Filed: **Mar. 6, 2013**

(86) PCT No.: **PCT/US2013/029375**
§ 371 (c)(1),
(2) Date: **Sep. 4, 2014**

(87) PCT Pub. No.: **WO2013/134388**
PCT Pub. Date: **Sep. 12, 2013**

(65) **Prior Publication Data**

US 2015/0081110 A1 Mar. 19, 2015

Related U.S. Application Data

(60) Provisional application No. 61/607,092, filed on Mar. 6, 2012.

(51) **Int. Cl.**
G05D 19/02 (2006.01)

(52) **U.S. Cl.**
CPC ***G05D 19/02*** (2013.01)

(58) **Field of Classification Search**
USPC 700/282
See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

2,447,230 A 8/1948 Brown
3,578,102 A 5/1971 Ross et al.

(Continued)

FOREIGN PATENT DOCUMENTS

EP 1226852 A2 7/2002
JP 03-113337 A 5/1991

(Continued)

OTHER PUBLICATIONS

Amemiya et al., Kinesthetic Illusion of Being Pulled Sensation Enables Haptic Navigation for Broad Social Applications 403 X Kinesthetic Illusion of Being Pulled Sensation Enables Haptic Navigation for Broad Social Applications, Apr. 1, 2010, XP55134361, Retrieved from the Internet: <URL:http:jjcdn. intechopen.comjpdfs-wm/9918.pdf>, [retrieved on Aug. 12, 2014].

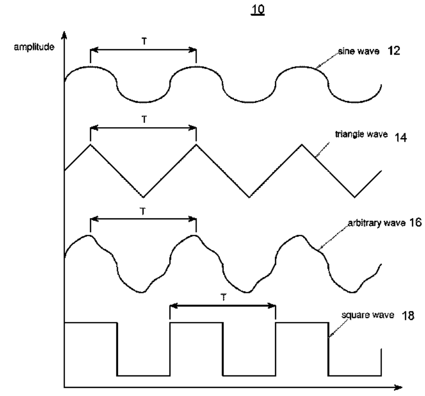
(Continued)

Primary Examiner — Kidest Bahta
(74) *Attorney, Agent, or Firm* — Lerner, David, Littenberg, Krumholz & Mentlik, LLP

(57) **ABSTRACT**

The disclosure relates to a Synchronized Array of Vibration Actuators in a Network Topology providing synchronized arrays of low-cost, readily available vibration actuators to emulate superlative single actuators and bring together sets of these emulated high-performance actuators to create a broad range of desired control effects. Such actuator arrays may operate in both spatial and temporal modes, creating haptic effects that relate to the user via their position and orientation in space. The spatial mode may create h-pulses or the amplitude of a vibrational effect may change based on position of the device. The temporal mode may create vibrational effects that interact with the user to create an awareness of time. Additionally modes include performance, bandwidth, magnitude and reliability modes. The different control modalities may be combined together into a single vector control space, which spans the haptic capabilities of sets and/or subsets of actuators.

32 Claims, 85 Drawing Sheets



(56) **References Cited**

U.S. PATENT DOCUMENTS

3,984,805 A	10/1976	Silverman	
4,098,133 A	7/1978	Frische et al.	
4,710,656 A	12/1987	Studer	
4,749,891 A	6/1988	Sheng	
4,788,968 A	12/1988	Rudashevsky et al.	
4,982,374 A	1/1991	Edington et al.	
5,136,194 A	8/1992	Oudet et al.	
5,327,120 A	7/1994	McKee et al.	
5,589,828 A	12/1996	Armstrong	
5,643,087 A	7/1997	Marcus et al.	
5,666,138 A	9/1997	Culver	
5,780,948 A	7/1998	Lee et al.	
5,825,663 A	10/1998	Barba et al.	
5,857,986 A	1/1999	Moriyasu	
5,903,077 A	5/1999	Garnjost et al.	
5,923,317 A	7/1999	Sayler et al.	
5,929,607 A	7/1999	Rosenberg et al.	
6,002,184 A	12/1999	Delson et al.	
6,009,986 A	1/2000	Bansemir et al.	
6,023,116 A	2/2000	Kikuchi et al.	
6,147,422 A	11/2000	Delson et al.	
6,208,497 B1	3/2001	Seale et al.	
6,236,125 B1	5/2001	Oudet et al.	
6,275,213 B1	8/2001	Tremblay et al.	
6,307,285 B1	10/2001	Delson et al.	
6,389,941 B1	5/2002	Michler	
6,397,285 B1	5/2002	Hashimoto et al.	
6,424,333 B1	7/2002	Tremblay et al.	
6,504,278 B1	1/2003	Bald et al.	
6,618,646 B1	9/2003	Dyer	
6,641,480 B2	11/2003	Murzanski et al.	
6,679,776 B1	1/2004	Nishiumi et al.	
6,693,622 B1	2/2004	Shahoian et al.	
6,704,001 B1	3/2004	Schena et al.	
6,742,960 B2	6/2004	Corcoran et al.	
6,809,727 B2	10/2004	Piot et al.	
6,824,468 B2	11/2004	Murzanski et al.	
6,864,877 B2	3/2005	Braun et al.	
6,873,067 B2	3/2005	Ichii et al.	
6,883,373 B2	4/2005	Dyer	
6,982,696 B1	1/2006	Shahoian	
6,992,462 B1	1/2006	Hussaini et al.	
7,084,854 B1	8/2006	Moore et al.	
7,091,948 B2	8/2006	Chang et al.	
7,182,691 B1	2/2007	Schena	
7,209,118 B2	4/2007	Shahoian et al.	
7,218,310 B2	5/2007	Tierling et al.	
7,315,098 B2	1/2008	Kunita et al.	
7,446,752 B2	11/2008	Goldenberg et al.	
7,561,142 B2	7/2009	Shahoian et al.	
7,683,508 B2	3/2010	Delson	
7,916,576 B2	3/2011	Beasley et al.	
7,919,945 B2	4/2011	Houston et al.	
7,931,605 B2	4/2011	Murison	
8,384,316 B2	2/2013	Houston et al.	
8,981,682 B2 *	3/2015	Delson	A63F 13/06 318/114
2002/0024503 A1	2/2002	Armstrong	
2002/0080112 A1	6/2002	Braun et al.	
2003/0030619 A1 *	2/2003	Martin	A63F 13/06 345/156
2003/0038774 A1	2/2003	Piot et al.	
2004/0108992 A1	6/2004	Rosenberg	
2004/0164959 A1	8/2004	Rosenberg et al.	
2004/0183782 A1	9/2004	Shahoian et al.	
2004/0227727 A1	11/2004	Schena et al.	
2004/0233161 A1	11/2004	Shahoian et al.	
2004/0233167 A1	11/2004	Braun et al.	
2005/0030284 A1 *	2/2005	Braun	G06F 3/016 345/156
2005/0052415 A1	3/2005	Braun et al.	
2005/0077845 A1	4/2005	Olgac et al.	
2005/0128186 A1	6/2005	Shahoian et al.	
2005/0134561 A1	6/2005	Tierling et al.	
2005/0134562 A1	6/2005	Grant et al.	
2005/0195168 A1	9/2005	Rosenberg et al.	
2005/0219206 A1	10/2005	Schena et al.	
2005/0221894 A1	10/2005	Lum et al.	
2005/0237314 A1	10/2005	Ryynanen	
2006/0015045 A1	1/2006	Zets et al.	
2006/0290662 A1	12/2006	Houston et al.	
2007/0091063 A1	4/2007	Nakamura et al.	
2007/0236450 A1	10/2007	Colgate et al.	
2008/0252594 A1	10/2008	Gregorio et al.	
2010/0245237 A1	9/2010	Nakamura	
2010/0253487 A1	10/2010	Grant et al.	
2011/0050404 A1	3/2011	Nakamura et al.	
2011/0124959 A1	5/2011	Murison	
2012/0232780 A1 *	9/2012	Delson	A63F 13/06 701/400

FOREIGN PATENT DOCUMENTS

JP	2000-333464 A	11/2000
JP	2001-212508 A	8/2001
JP	2002-205008 A	7/2002
JP	2003-199974 A	7/2003
JP	2003-345233 A	12/2003
JP	2004-174309 A	6/2004
JP	2012125135 A	6/2012
KR	10-2010-0029158 A	3/2010
KR	20100044381 A	4/2010
KR	10-2010-0125941 A	12/2010
WO	9318475 A1	9/1993

OTHER PUBLICATIONS

Extended European Search Report for application No. EP12757425 dated Oct. 30, 2014.

International Seach Report and Written Opinion for Application No. PCT/US2014/045984 dated Nov. 19, 2014.

Tappeiner et al., "Good vibrations: Asymmetric vibrations for directional haptic cues", Eurohaptics Conference, 2009 and Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems, World Haptics 2009, Third Joint, IEEE, Piscataway, NJ, USA, Mar. 18, 2009, pp. 285-289, XP031446857.

U.S. Appl. No. 61/844,100, filed Jul. 9, 2013.

U.S. Appl. No. 61/453,739, filed Mar. 17, 2011.

U.S. Appl. No. 60/694,468, filed Jun. 27, 2005.

U.S. Appl. No. 61/511,268, filed Jul. 25, 2011.

U.S. Appl. No. 61/607,092, filed Mar. 6, 2012.

"Essential Atlas of Physics and Chemistry" Barons, pp. 26-27, 2003.

European Search Report, EP 11174517, dated Nov. 28, 2011.

Freeman, M., "3-D Vibration Test System: Powerful, Unusual, International", Test Engineering & Management, Aug./Sep. 1992, pp. 10-12.

Harman, C. et al, "Multi-Axis Vibration Reduces Test Time", Evaluation Engineering, Jun. 2006, pp. 1-7.

International Search Report and Written Opinion for Application No. PCT/US2012/029440 dated Oct. 24, 2012.

International Search Report and Written Opinion for Application No. PCT/US2013/029375 dated Jul. 22, 2013.

Maor, E., "Trigonometric Delights", Princeton University Press, Princeton, NJ, 1998, pp. 145-149.

NanoMuscle Linear Actuator, http://3w.gfec.com.tw/english/service/content/elinear_actuator.htm, printed Jan. 2, 2006.

Office Action from Japanese Application No. 2007-550426 dated Aug. 9, 2011.

Office Action from Japanese Application No. 2008-519530 dated Aug. 2, 2011.

Supplementary European Search Report, EP 06774221, dated Jun. 19, 2009.

U.S. Appl. No. 11/325,036, filed Jan. 4, 2006.

U.S. Appl. No. 11/476,436, filed Jun. 27, 2006.

Wyle Laboratories, "Multi Axis-Dynamic Vibration System", 2006.

Grunwald, Martin, "Human Haptic Perception: Basics and Applications", published by Birkhauser Verlag, (copyright 2008), 687 pages.

(56) **References Cited**

OTHER PUBLICATIONS

Albertos et al., "Feedback and Control for Everyone", published by Springer-Verlag, (copyright 2010), 337 pages.
Kern, Thorsten A., "Engineering Haptic Devices: A Beginner's Guide for Engineers", published by Springer-Verlag, copyright 2009, 504 pages.
"DRV8601 Haptic Driver for DC Motors (ERMs) and Linear Vibrators (LRAs) with Ultra-Fast Turnon", Texas Instruments Inc., SLOS629C, copyright 2010-2016, 25 pages.
"Haptics in Touchscreen Hand-Held Devices", Immersion Corporation, Apr. 2012, 12 pages.
Neisstein, Eric W., "Lissajous Curve", from MathWorld—A Wolfram Web Resource, retrieved from the Internet Mar. 23, 2016, <http://mathworld.wolfram.com/LissajousCurve.html>, 2 pages.
Siegel, Eric, "Enabling high-definition haptics: introducing piezo actuators", The blogpost, Texas Instruments, retrieved from the Internet Mar. 23, 2016, <https://e2e.ti.com/blogs_/b/analogwire/archive/2011/07/25/enabling-high-definition-haptics-introducing-piezo-actuators>, 2 pages.
Precision Microdrives Limited: "App-bulletin 002: Discrete H-bridge Circuit for Enhanced Vibration Control", copyright 2011, 4 pages.
Precision Microdrives Limited, App-Bulletin-003: Driving Linear Resonance Vibration Actuators, copyright 2011, 10 pages.
"How to Disassemble an Xbox 360 Wireless Controller", retrieved from the Internet Mar. 23, 2016, <http://www.Instructables.com/id/How-To-Disassemble-an-Xbox-360-Wireless-Controller/>, 13 pages.
Todd et al., "Music and Connectionism", MIT Press, London, England, copyright 1991, 274 pages.
Kirk, Donald E, "Optimal Control Theory: an Introduction", Dover Publications, Inc., copyright 1998, 468 pages.
French, A. P., "Vibrations and Waves", published by W. W. Norton & Company Inc., copyright 1971 by The Massachusetts Institute of Technology, 325 pages.

* cited by examiner

FIG. 1 10

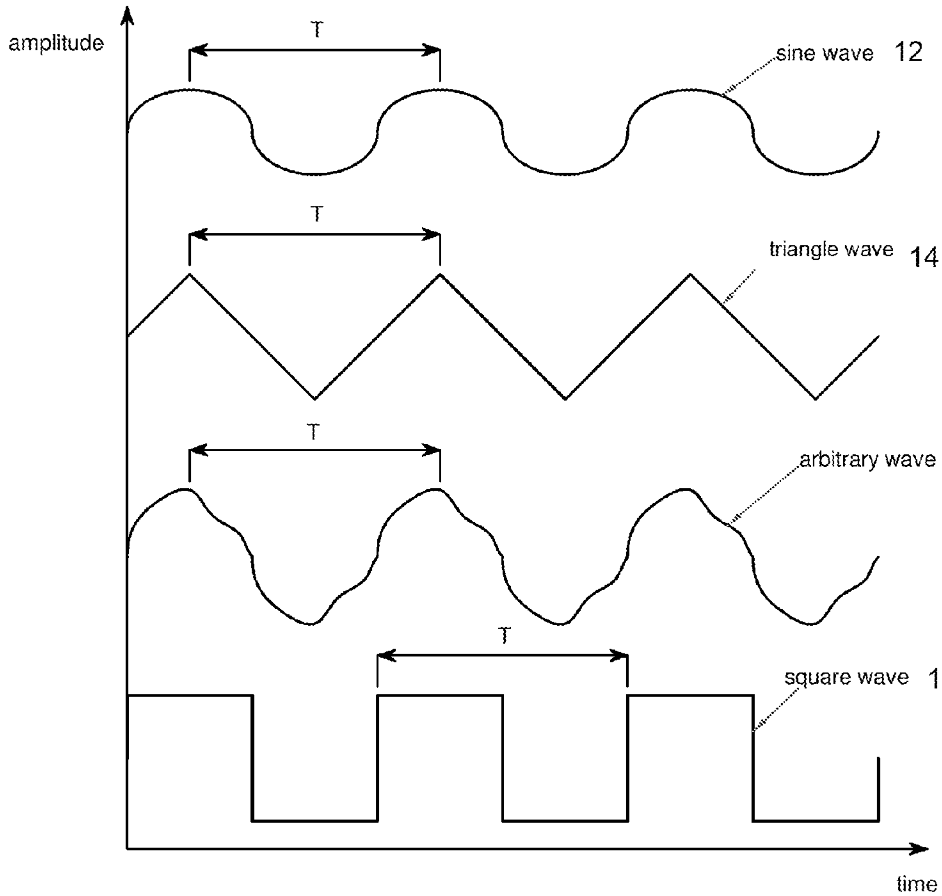


FIG. 2 20

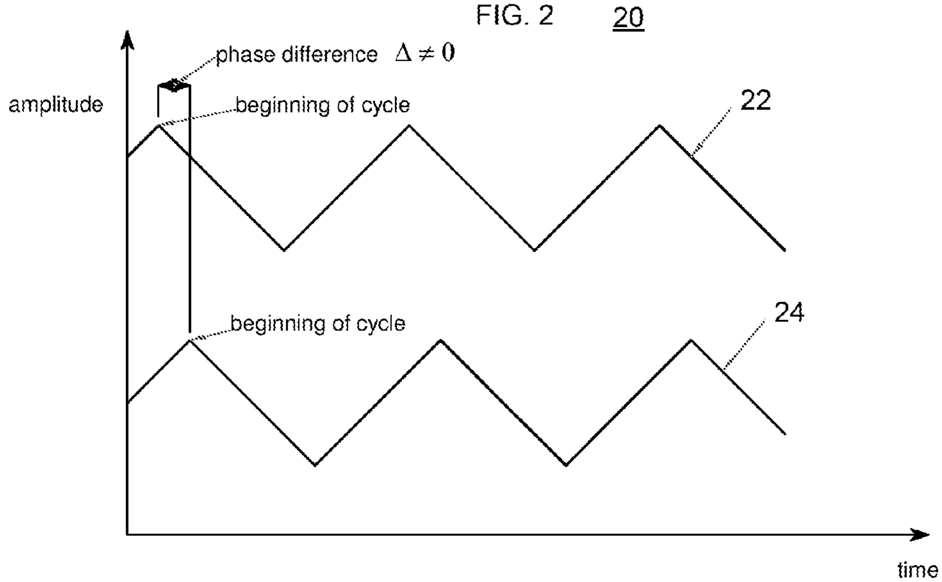


FIG. 3

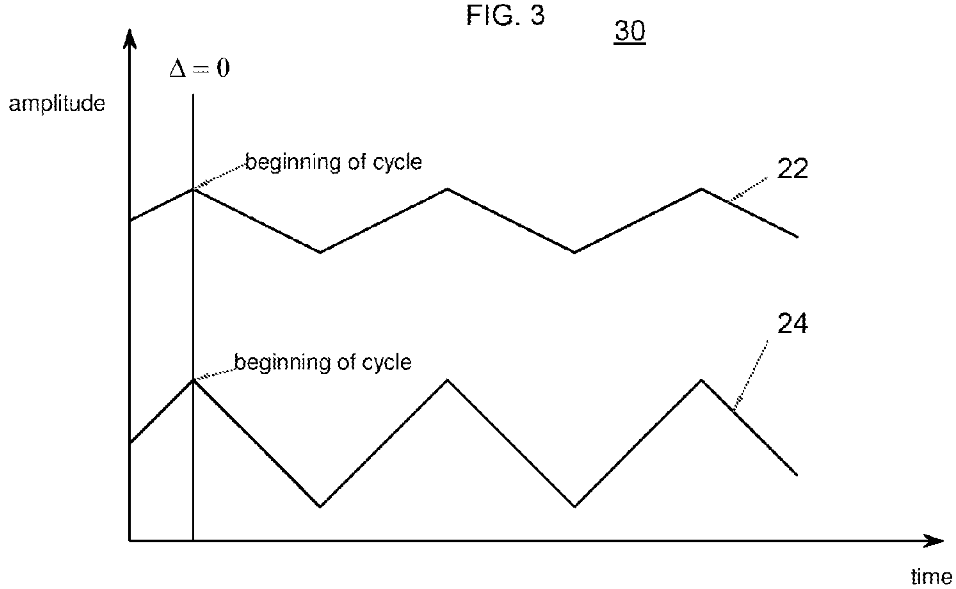


FIG. 4

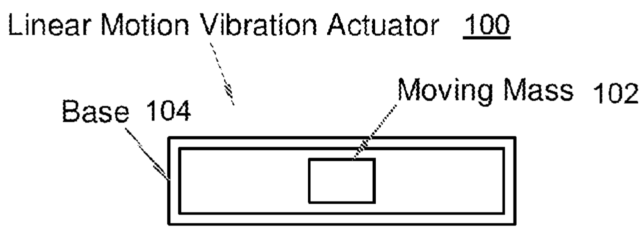


FIG. 5A

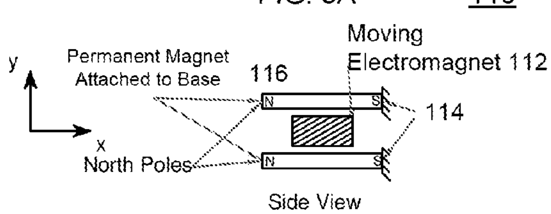


FIG. 5B

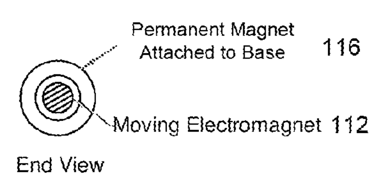


FIG. 6A 120 FIG. 6B

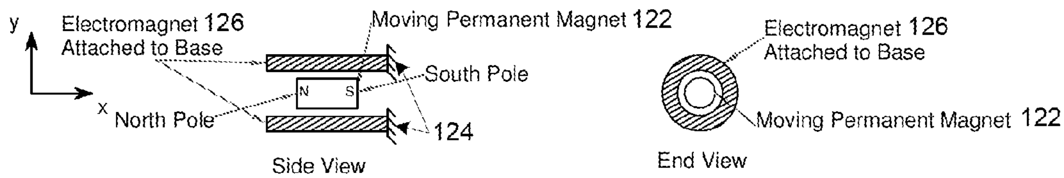


FIG. 7A 130 FIG. 7B

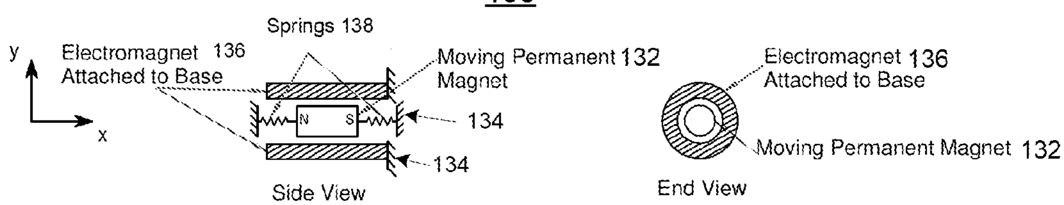


FIG. 8A 140 FIG. 8B

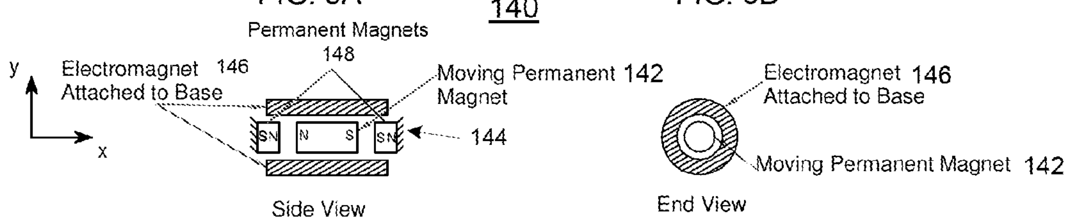


FIG. 9 150

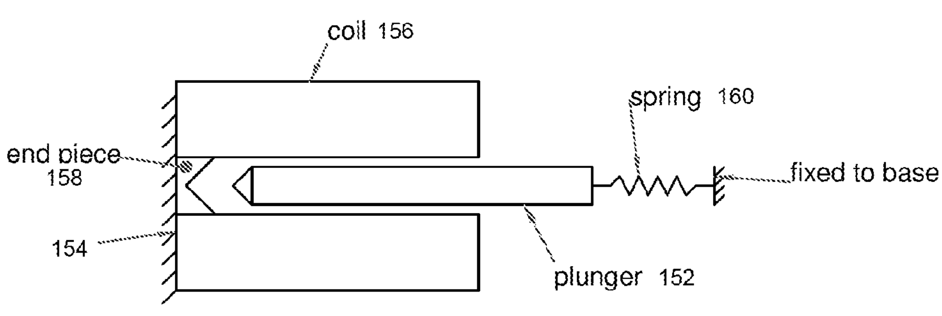


FIG. 10

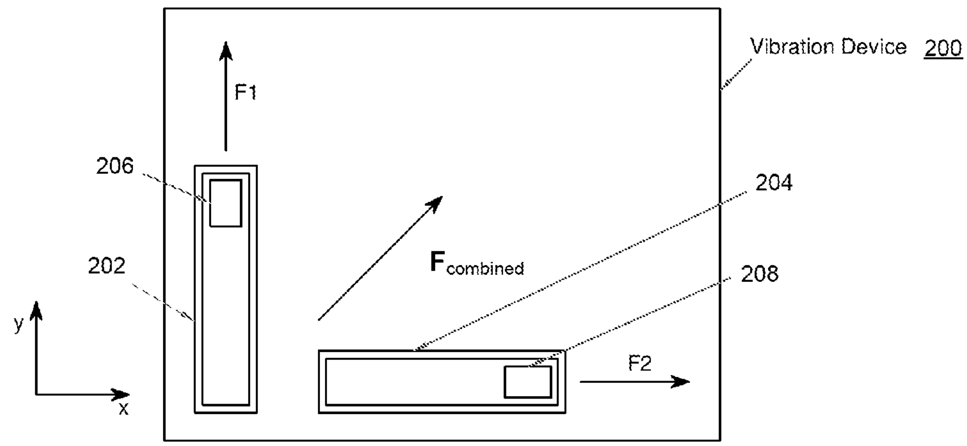

Vibration Device 200
F1
206
202
y
x
$F_{combined}$
204
208
F2

FIG. 11

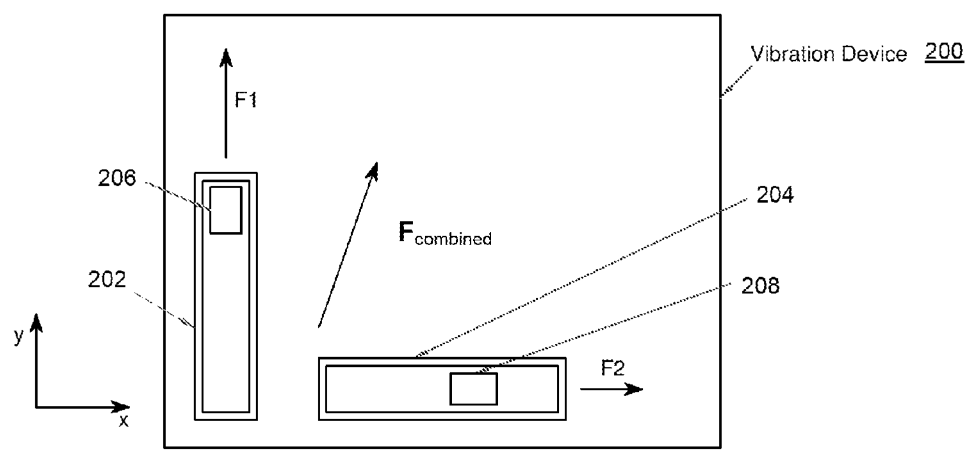

Vibration Device 200
F1
206
202
y
x
$F_{combined}$
204
208
F2

FIG. 12

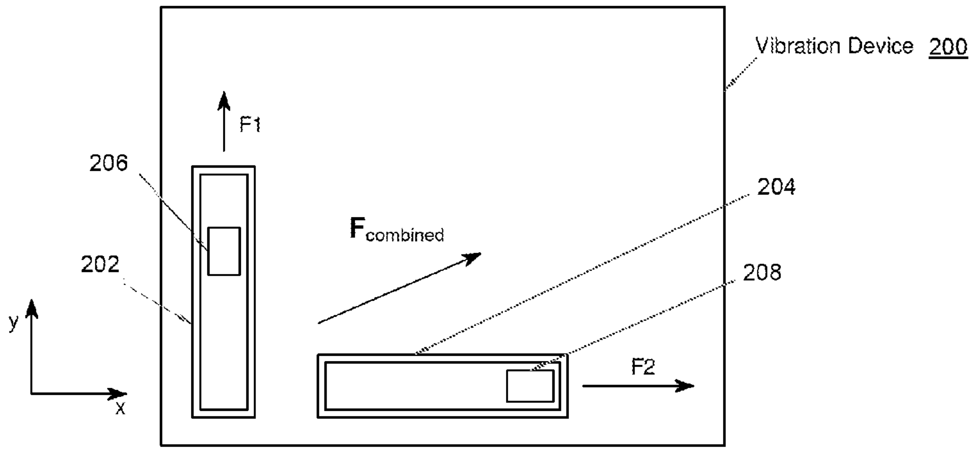


FIG. 13

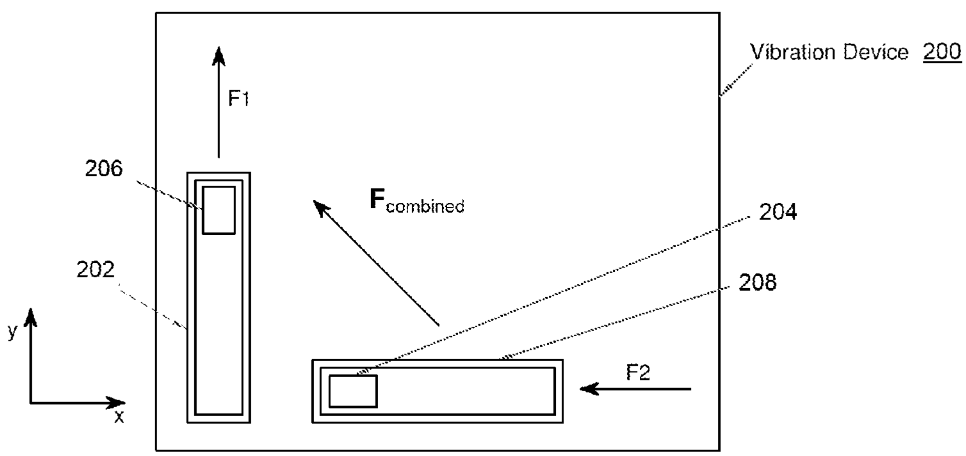


FIG. 14

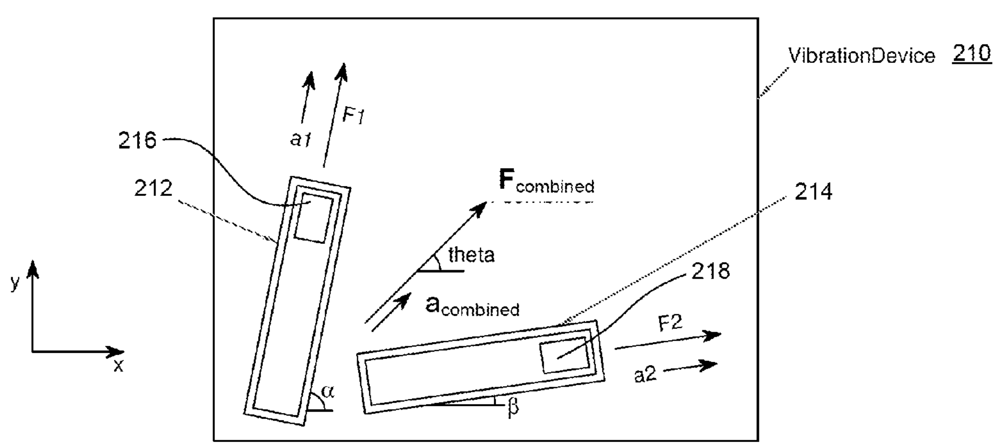


FIG. 15

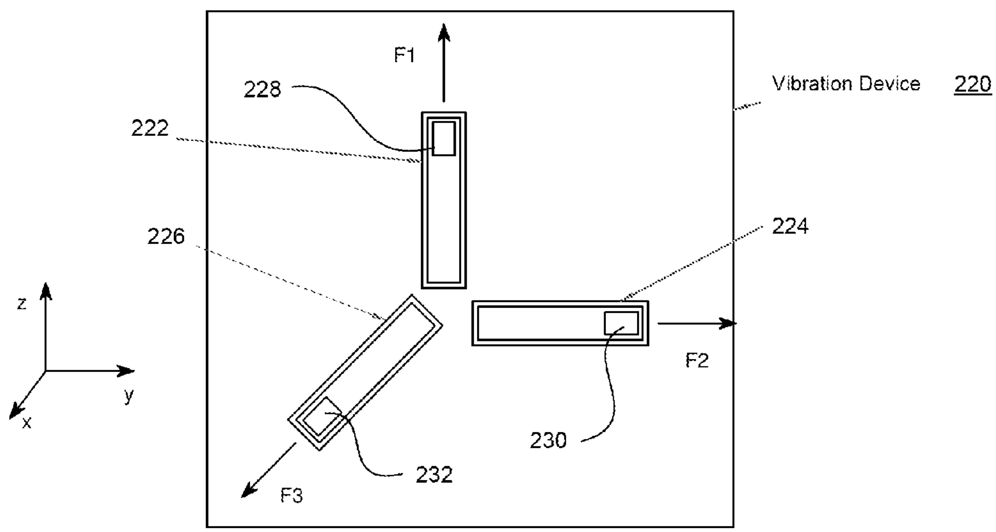

F1
228
222
Vibration Device 220
226
224
z
y
x
F2
230
232
F3

FIG. 16

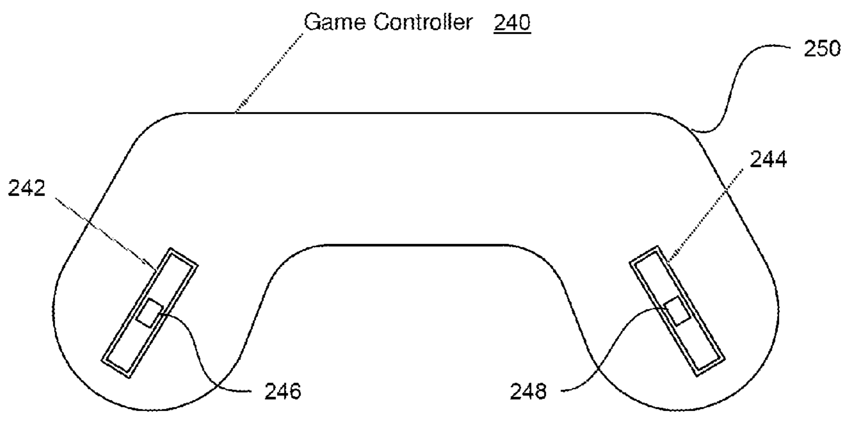

Game Controller 240
250
242
244
246
248

FIG. 17

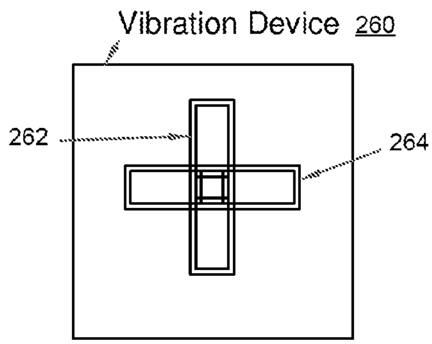


FIG. 18

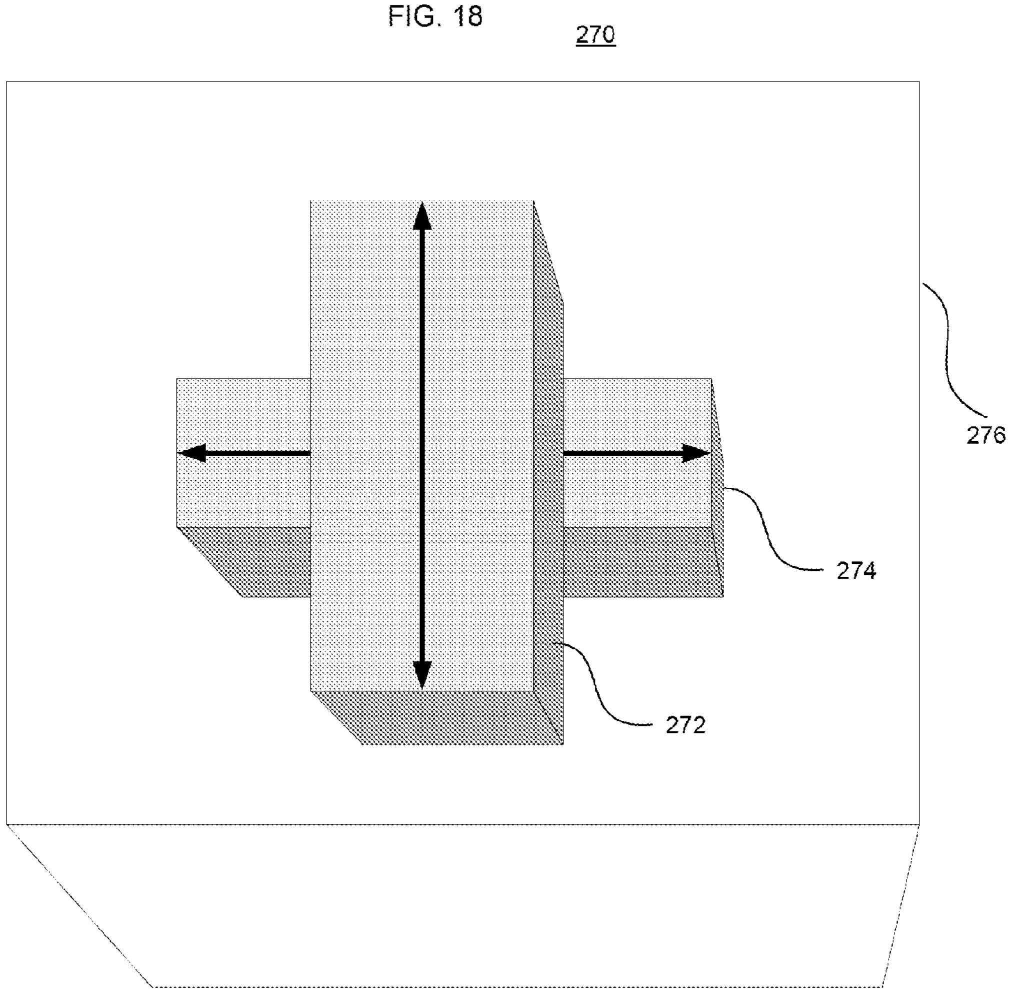


FIG. 19

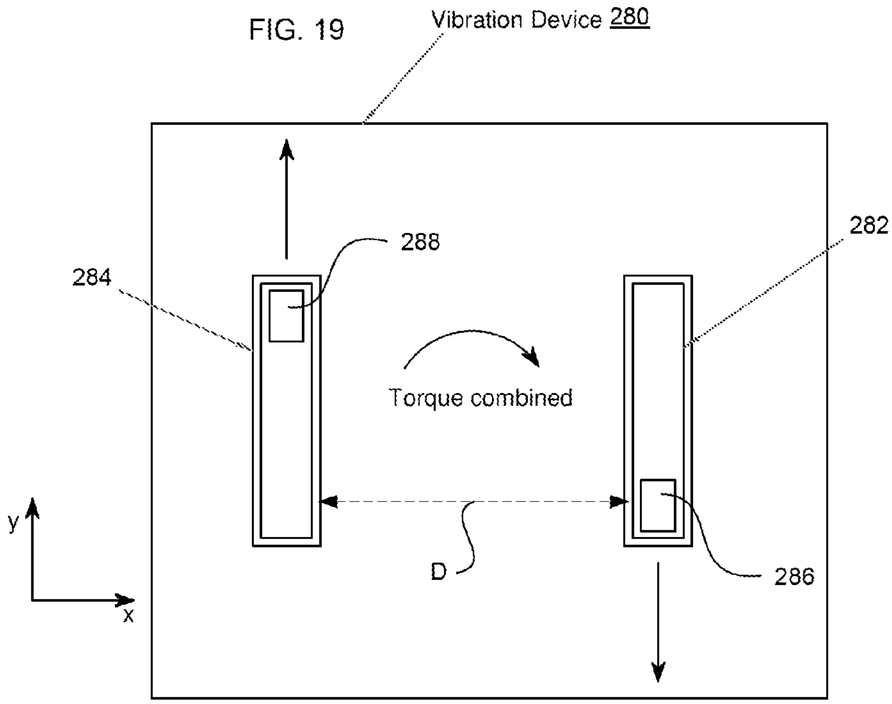

Vibration Device 280
288
282
284
Torque combined
y
x
D
286

FIG. 20

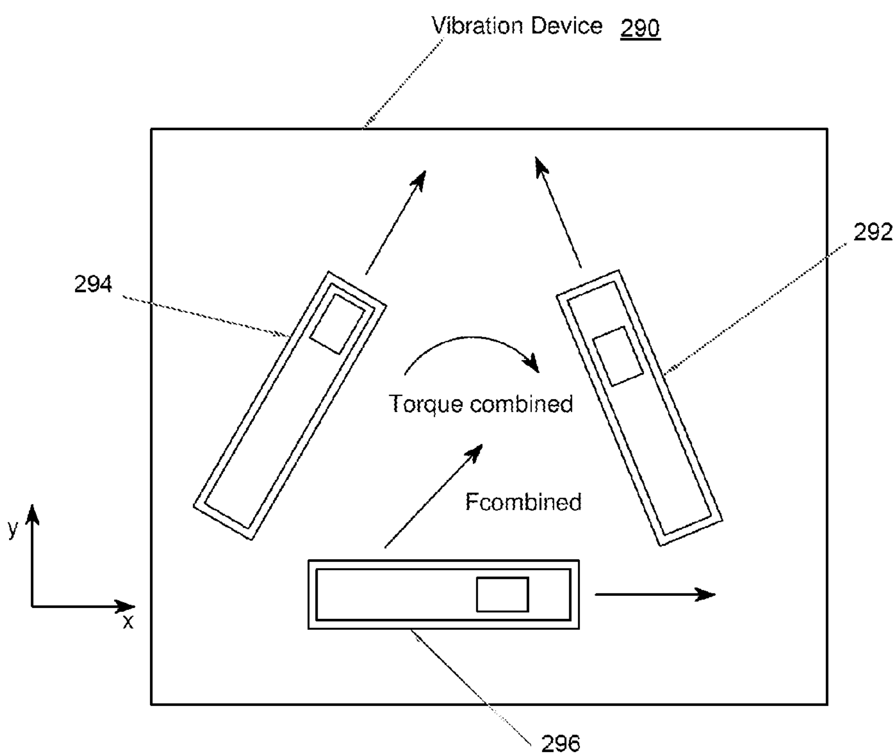

Vibration Device 290
292
294
Torque combined
Fcombined
y
x
296

FIG. 21 300

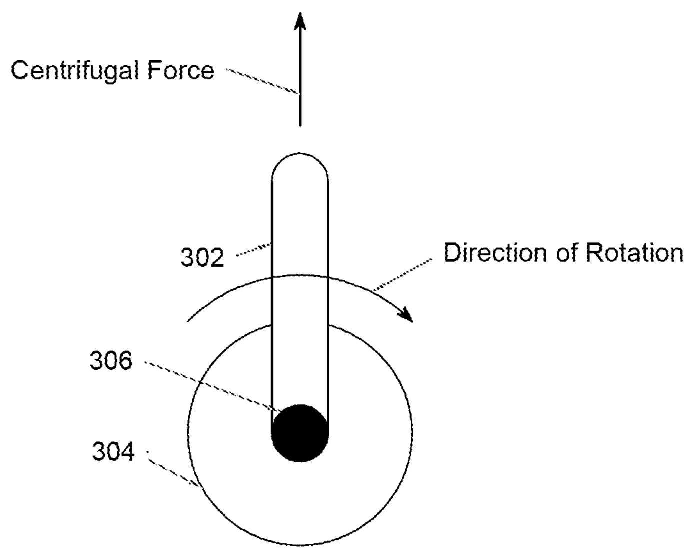


FIG. 22 310

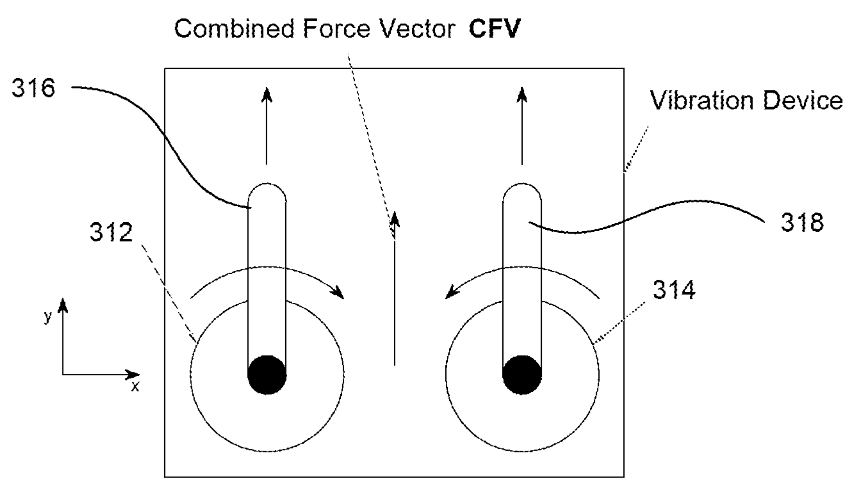


FIG. 23

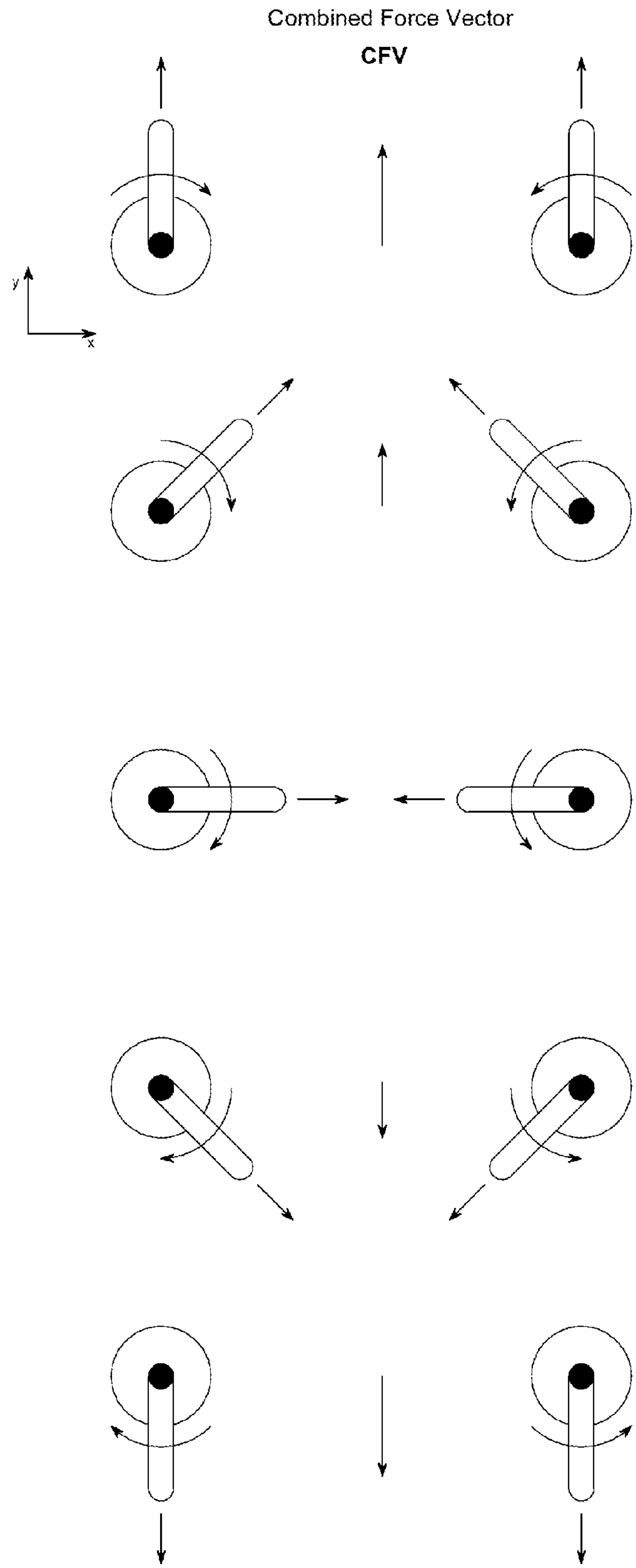


FIG. 24A

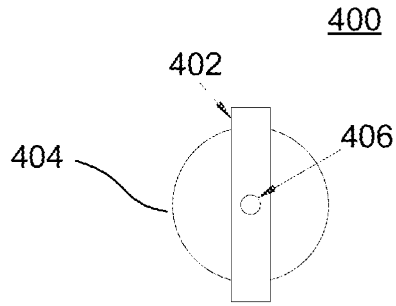


FIG. 24B

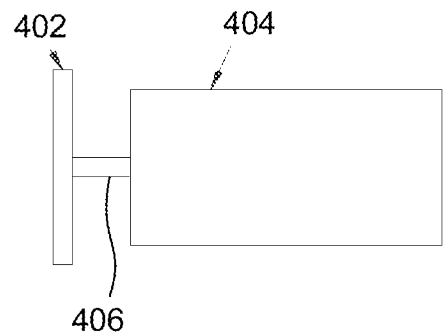


FIG. 24C

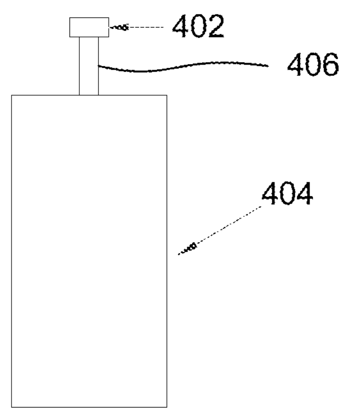


FIG. 25A 400

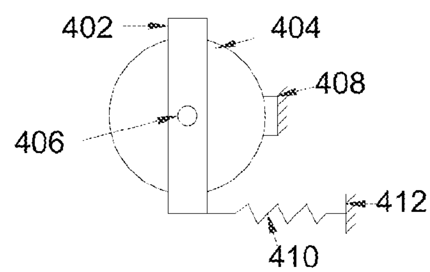
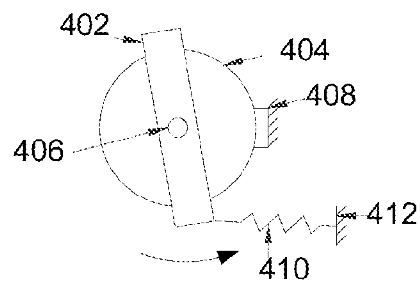
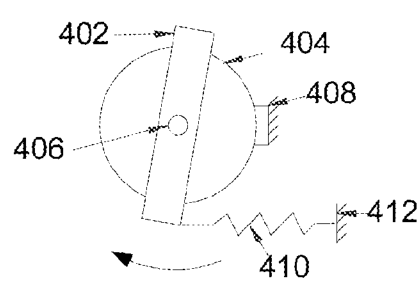


FIG. 25B



Pivoted Counterclockwise Position

FIG. 25C



Pivoted Clockwise Position

FIG. 26

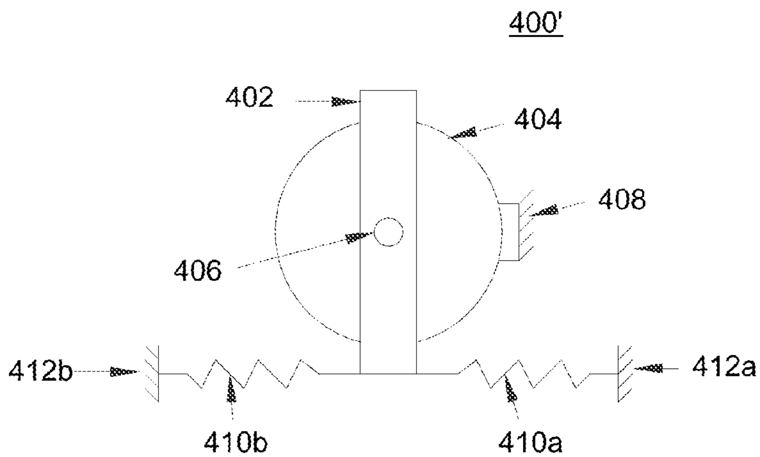

400'
402
404
408
406
412b
412a
410b
410a

FIG. 28

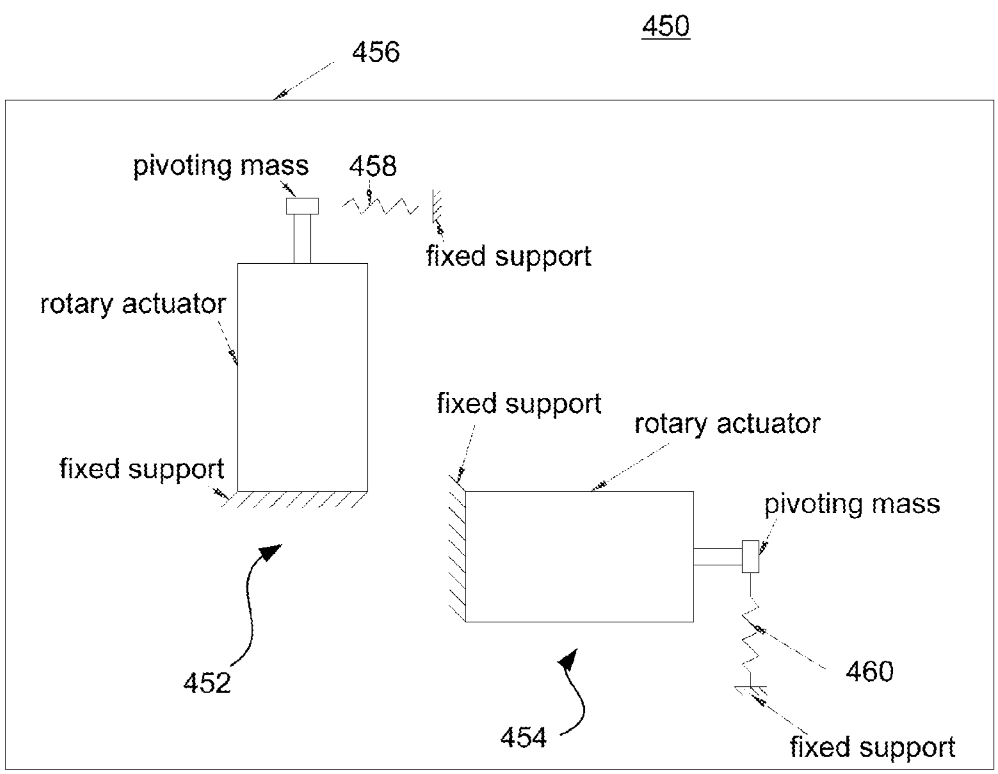

450
456
pivoting mass
458
fixed support
rotary actuator
fixed support
fixed support
rotary actuator
pivoting mass
452
454
460
fixed support

FIG. 27A

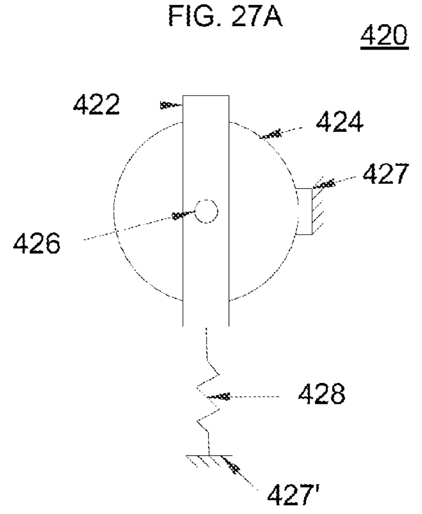


FIG. 27B

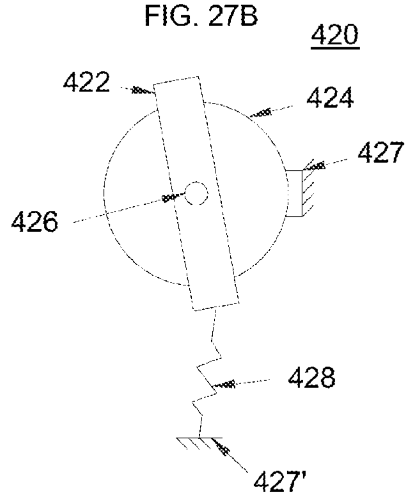


FIG. 27C

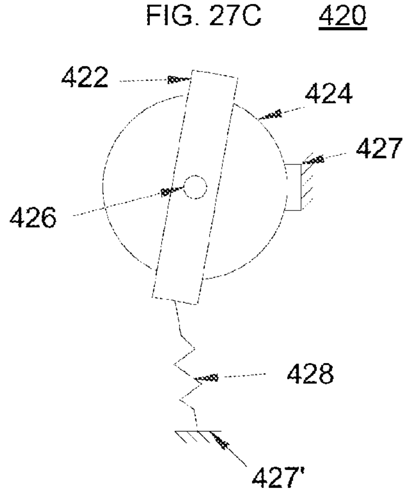


FIG. 27D

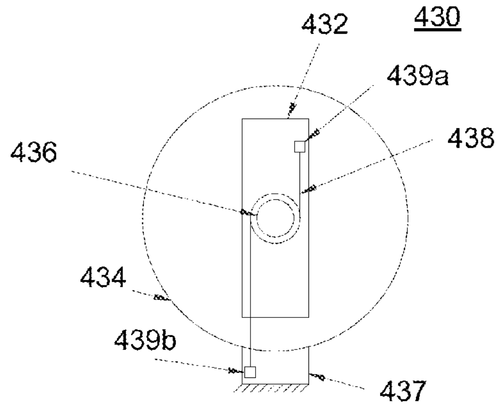


FIG. 27E

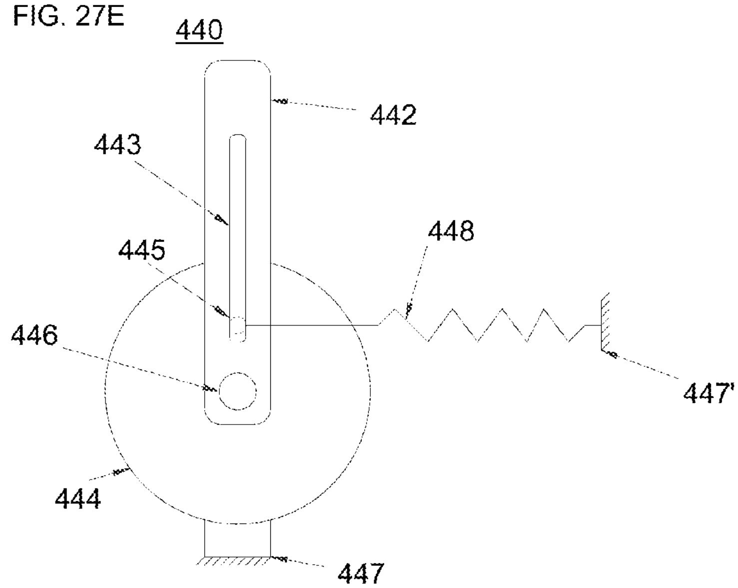


FIG. 27F

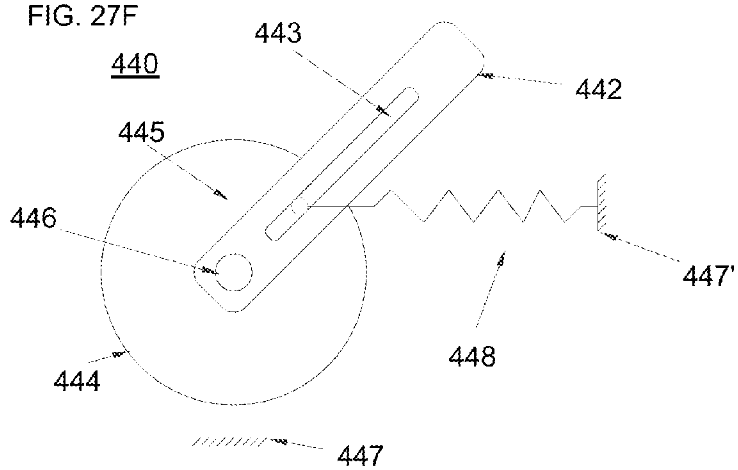


FIG. 29A

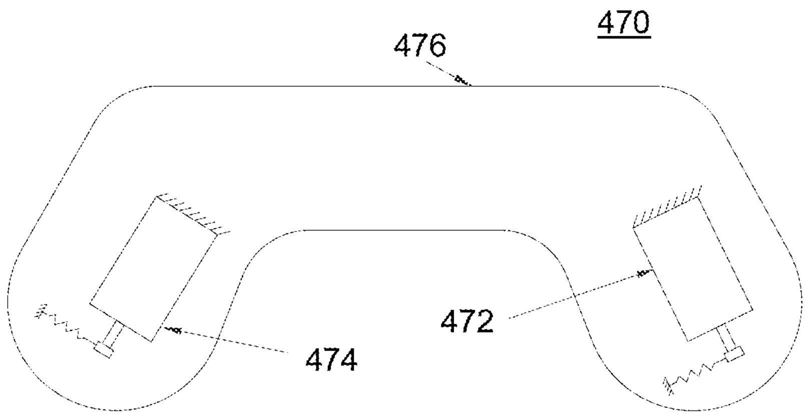


FIG. 29B

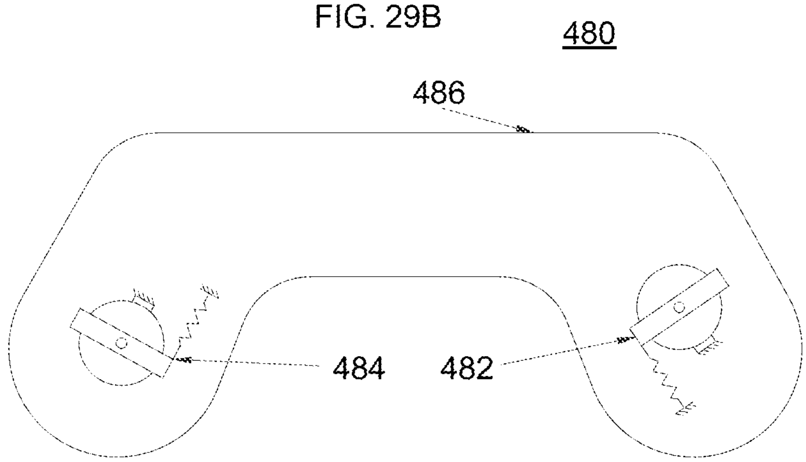


FIG. 30

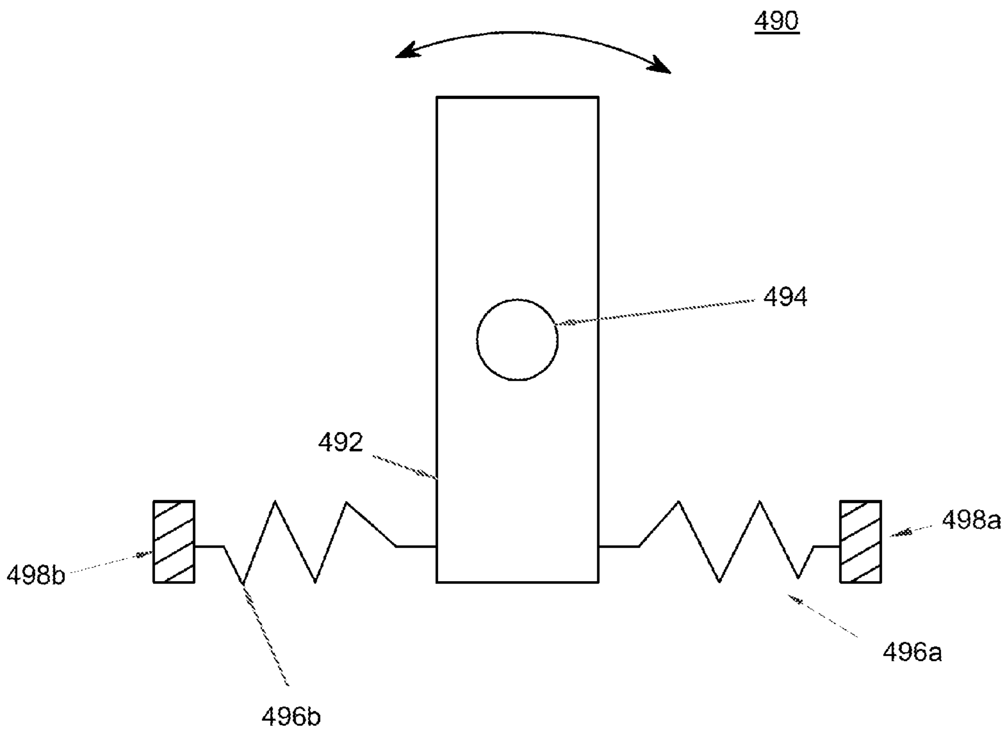


FIG. 31

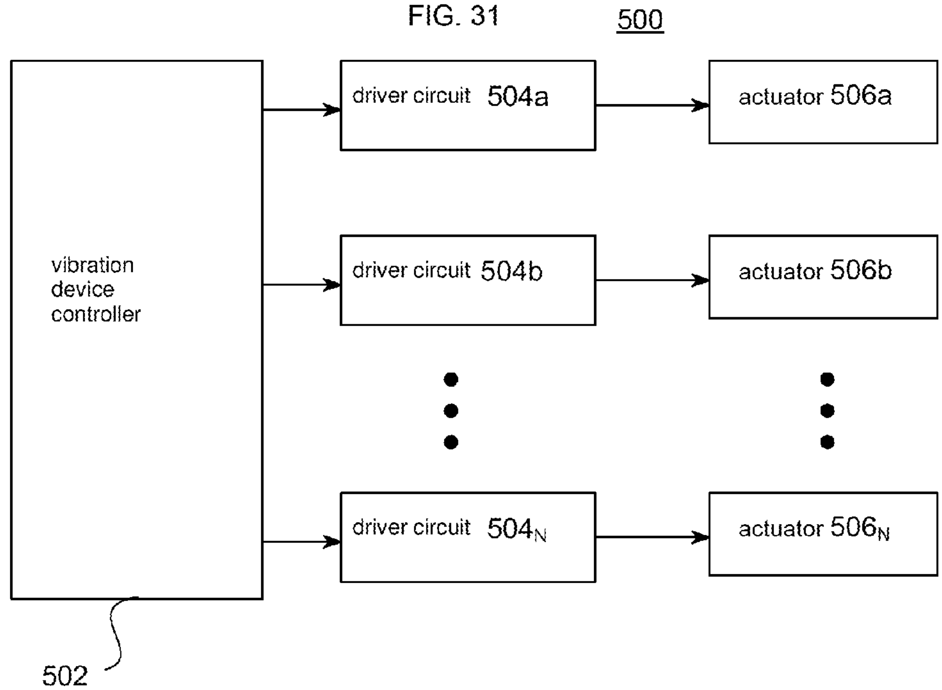


FIG. 32

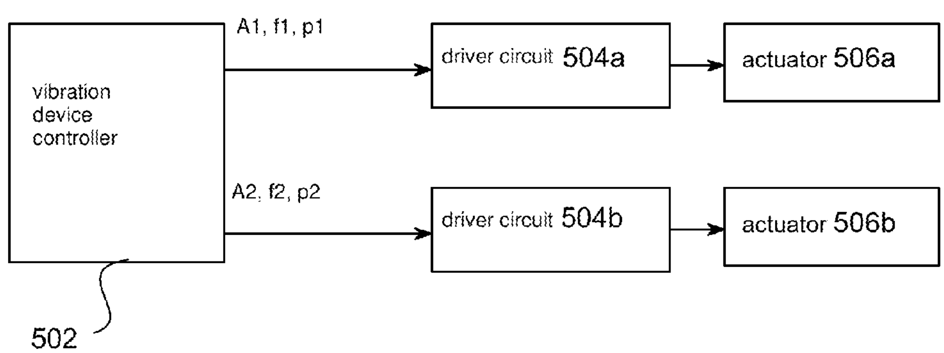


FIG. 33

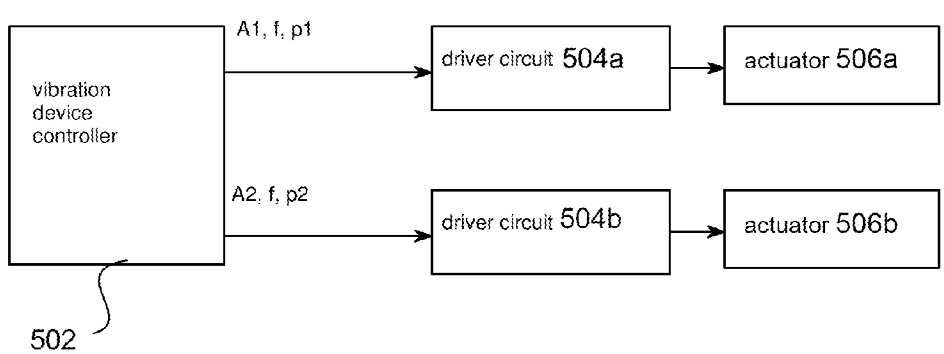


FIG. 34

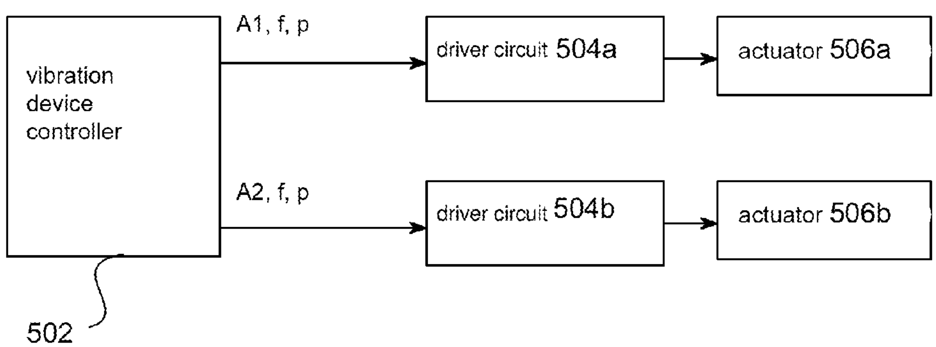

vibration
device
controller
A1, f, p
A2, f, p
driver circuit 504a
driver circuit 504b
actuator 506a
actuator 506b
502

FIG. 35

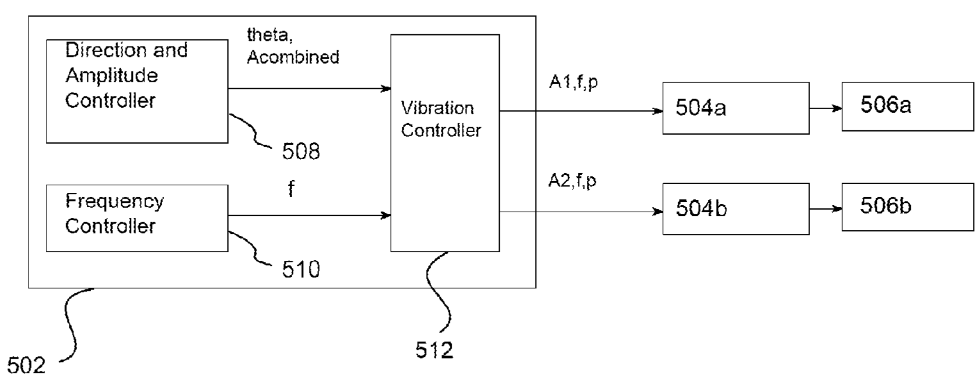

Direction and
Amplitude
Controller
theta,
Acombined
508
Frequency
Controller
f
510
Vibration
Controller
A1,f,p
A2,f,p
504a
506a
504b
506b
502
512

FIG. 36A

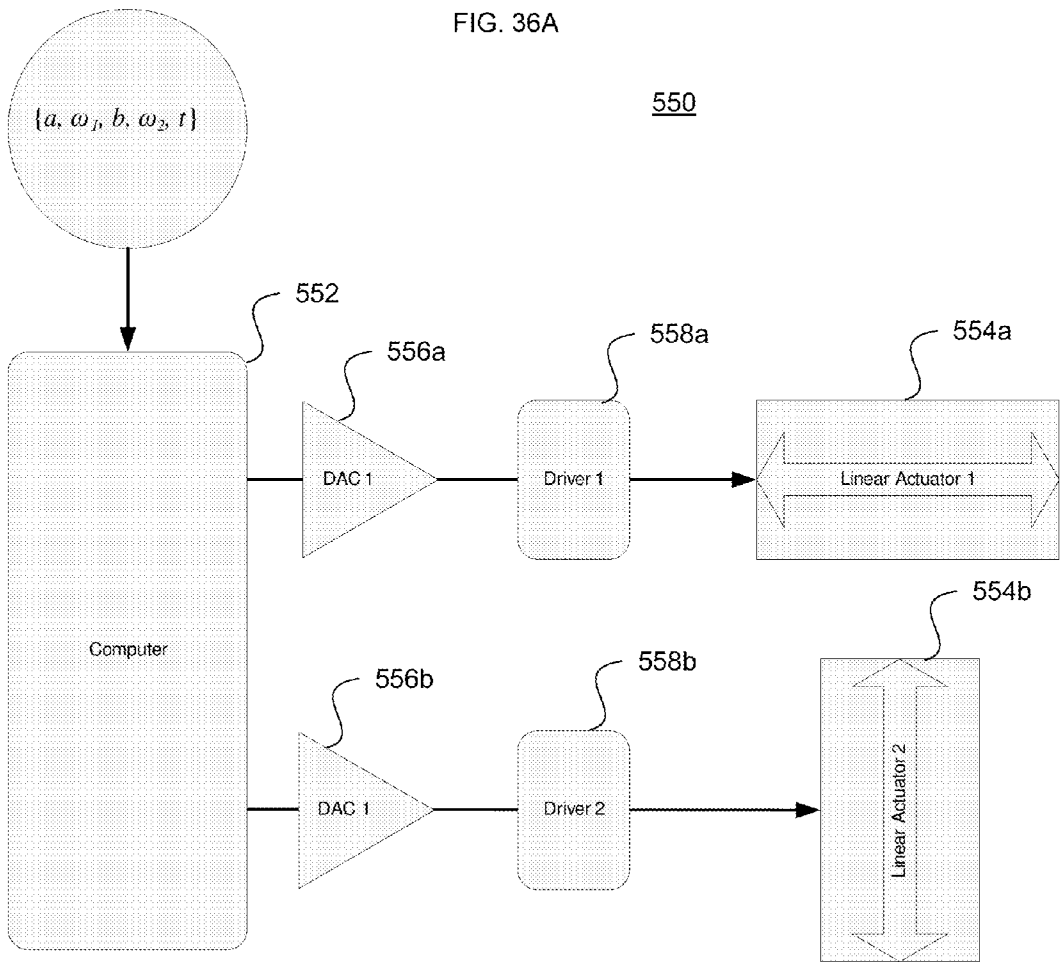

550
$\{a, \omega_1, b, \omega_2, t\}$
552
Computer
556a
DAC 1
558a
Driver 1
554a
Linear Actuator 1
556b
DAC 1
558b
Driver 2
554b
Linear Actuator 2

FIG. 36B

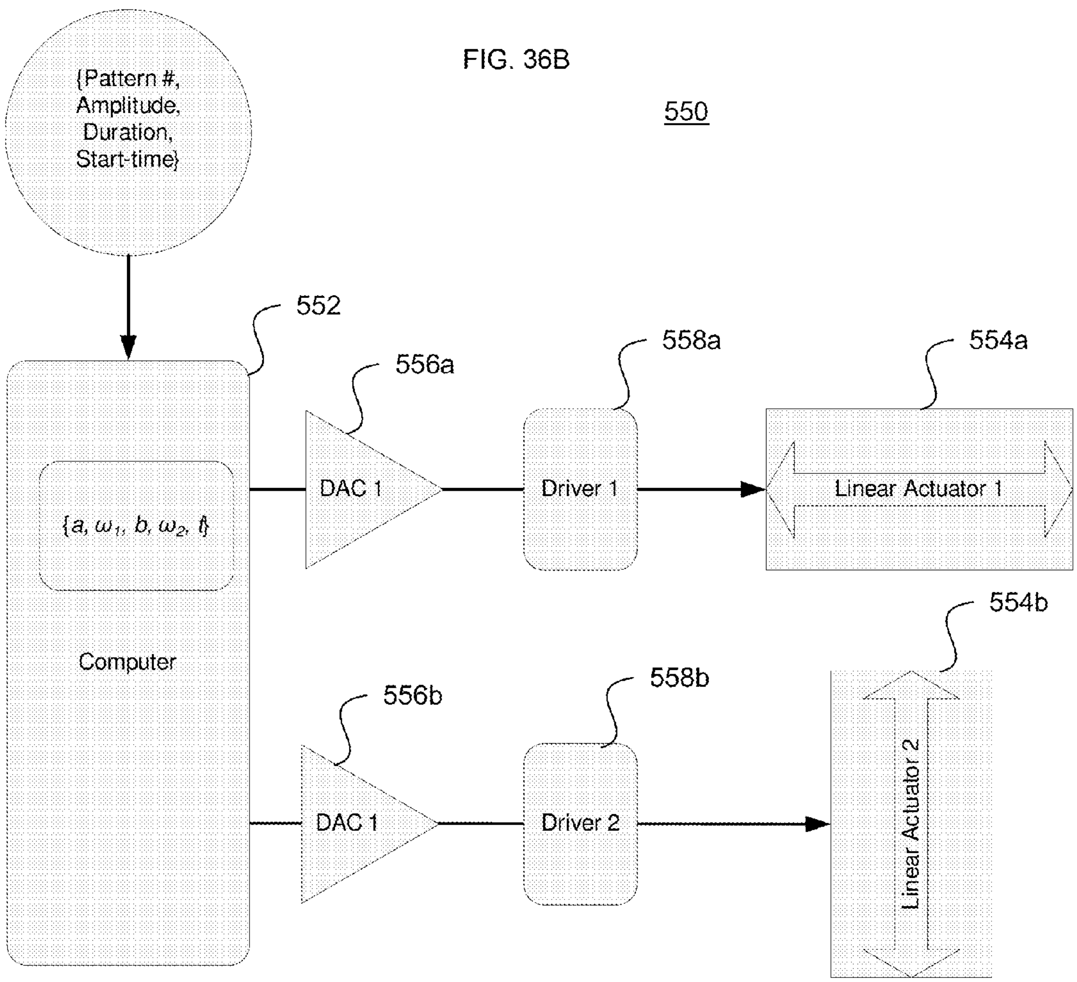

550
{Pattern #,
Amplitude,
Duration,
Start-time}
552
556a
558a
554a
$\{a, \omega_1, b, \omega_2, t\}$
DAC 1
Driver 1
Linear Actuator 1
554b
Computer
556b
558b
DAC 1
Driver 2
Linear Actuator 2

FIG. 37

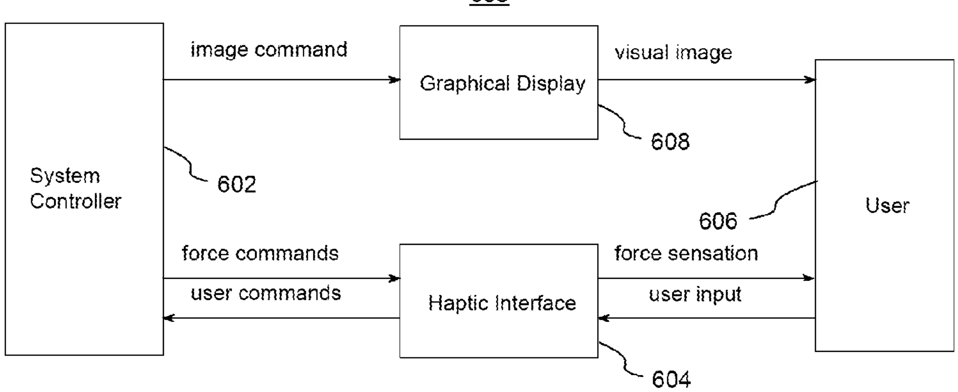


FIG. 38

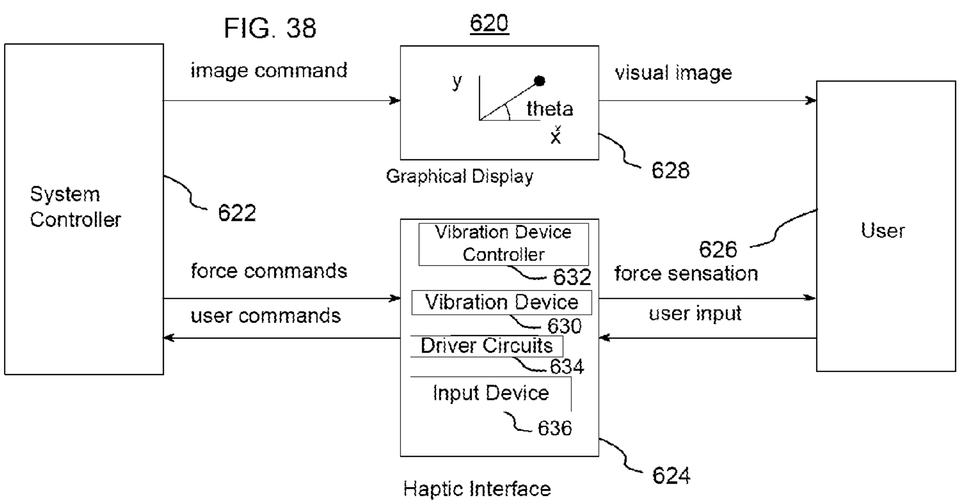


FIG. 39

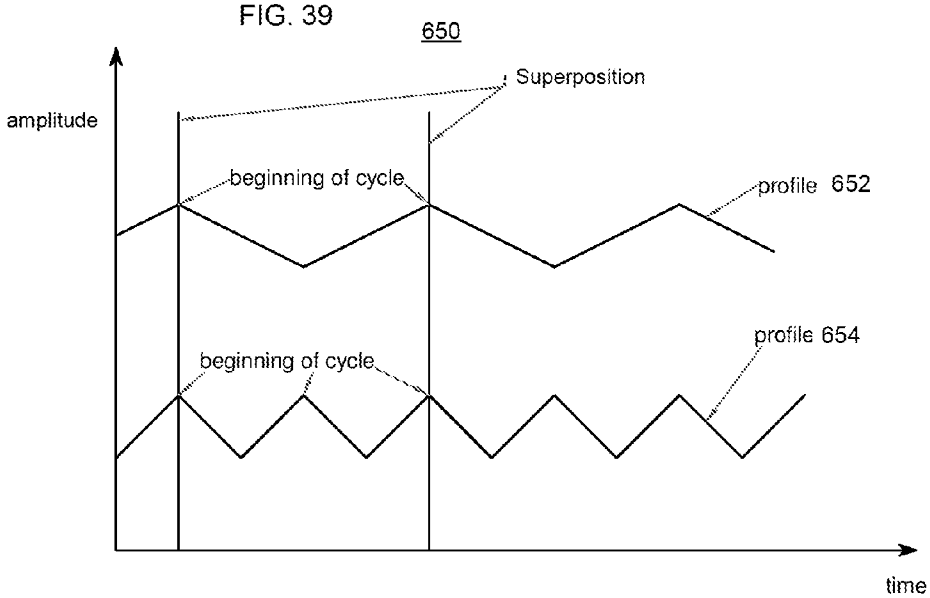


FIG. 40

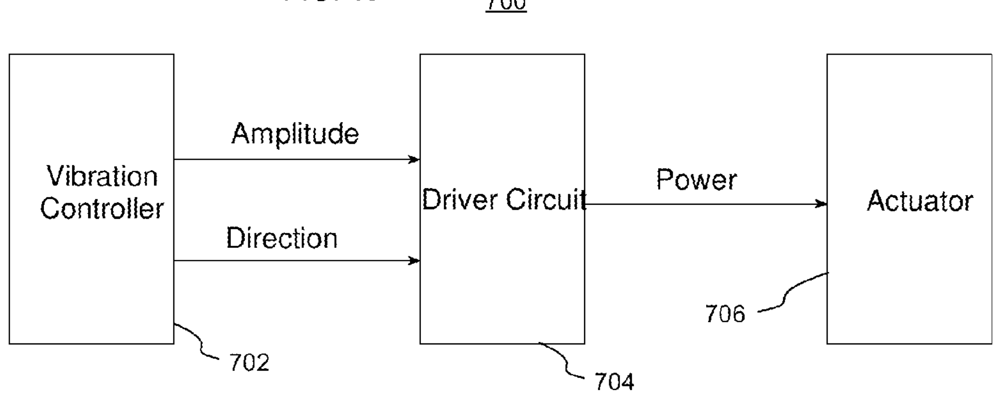


FIG. 41

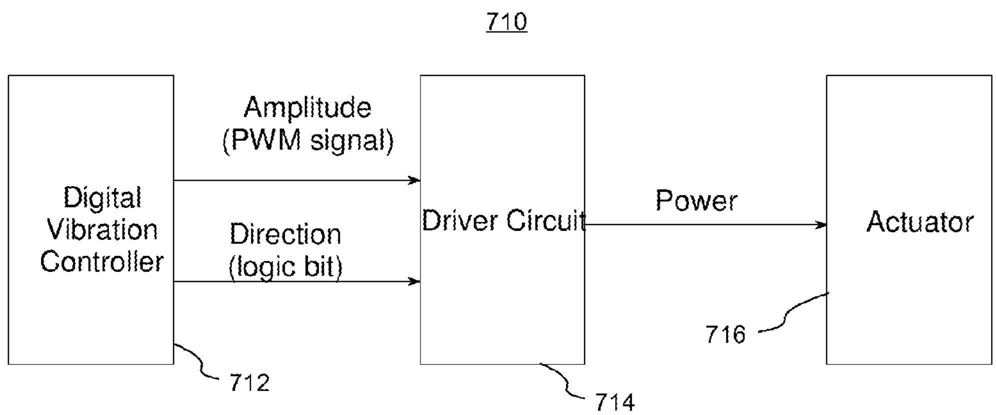


FIG. 42

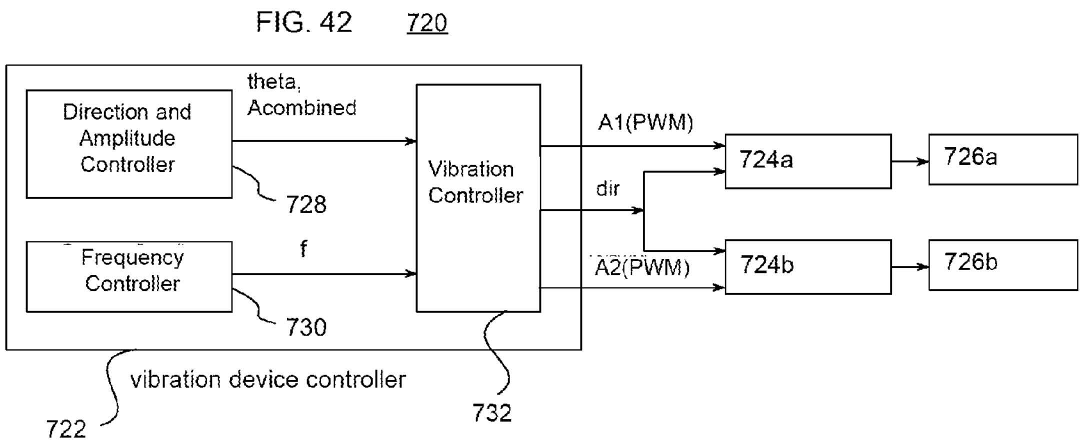


FIG. 43

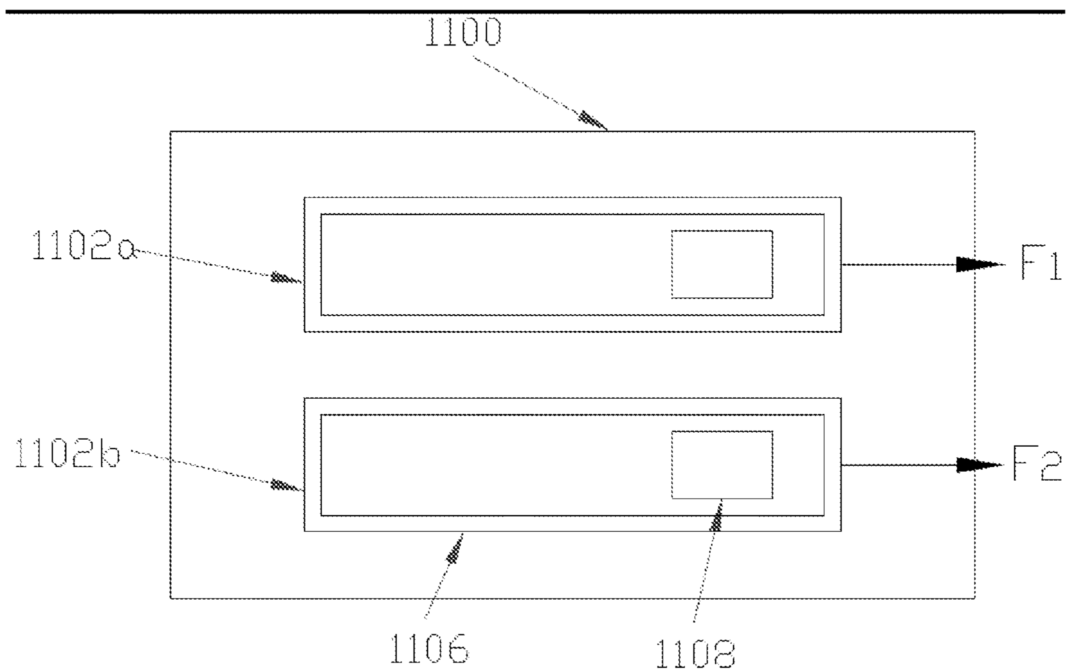


FIG. 44

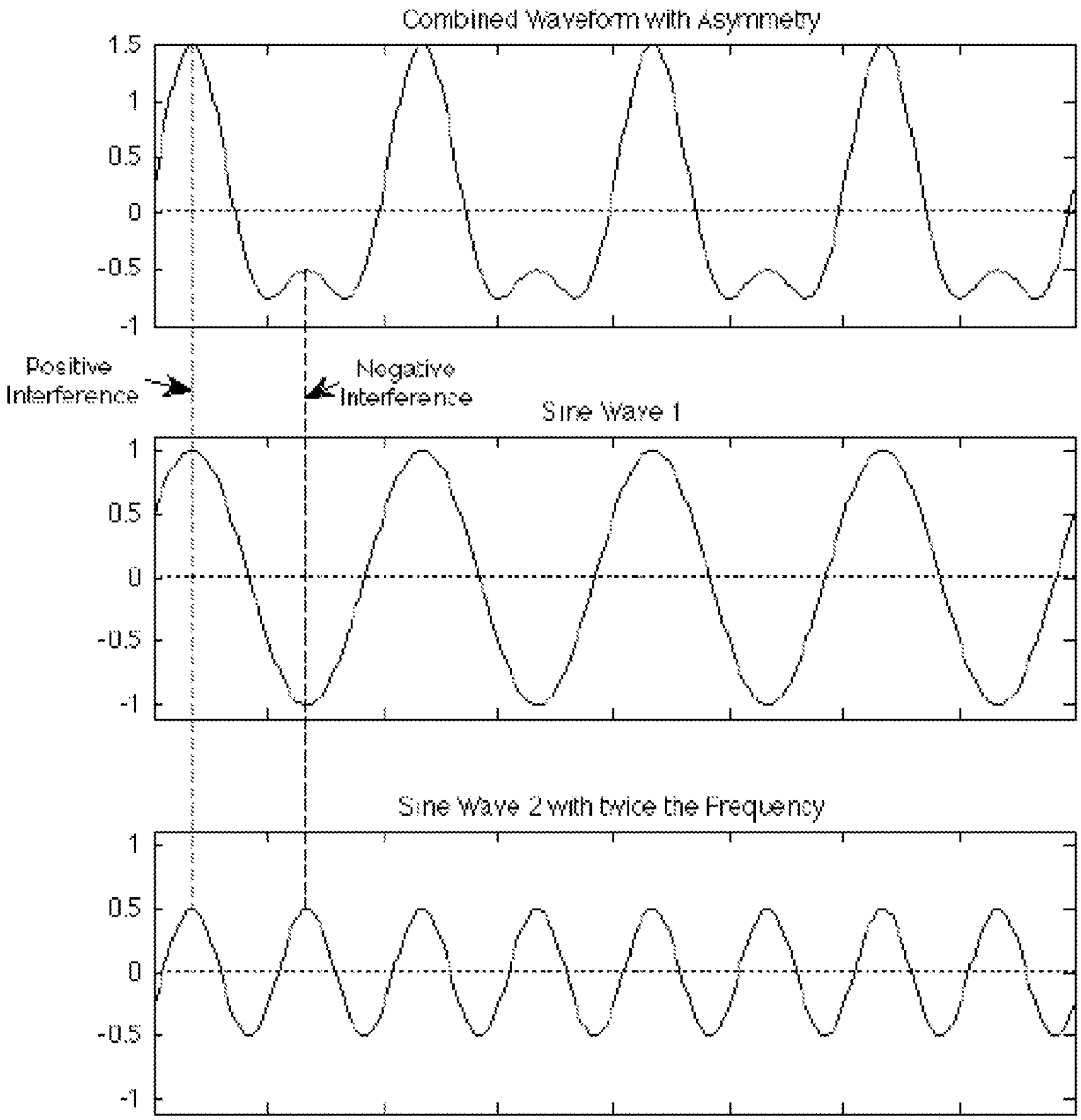

Combined Waveform with Asymmetry
1.5
1
0.5
0
-0.5
-1
Positive Interference
Negative Interference
Sine Wave 1
1
0.5
0
-0.5
-1
Sine Wave 2 with twice the Frequency
1
0.5
0
-0.5
-1

FIG. 45

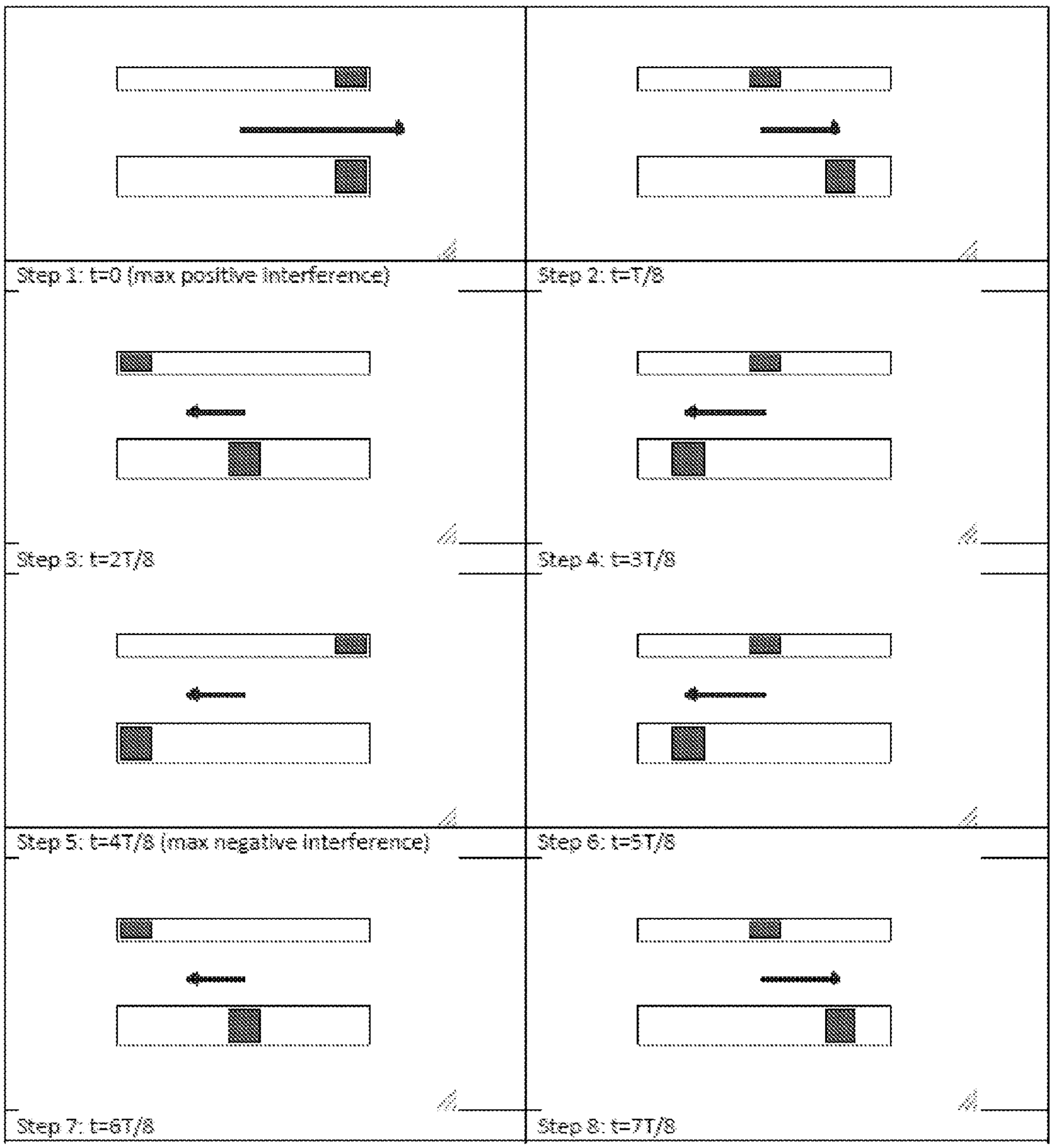

Step 1: t=0 (max positive interference)
Step 2: t=T/8
Step 3: t=2T/8
Step 4: t=3T/8
Step 5: t=4T/8 (max negative interference)
Step 6: t=5T/8
Step 7: t=6T/8
Step 8: t=7T/8

FIG. 46

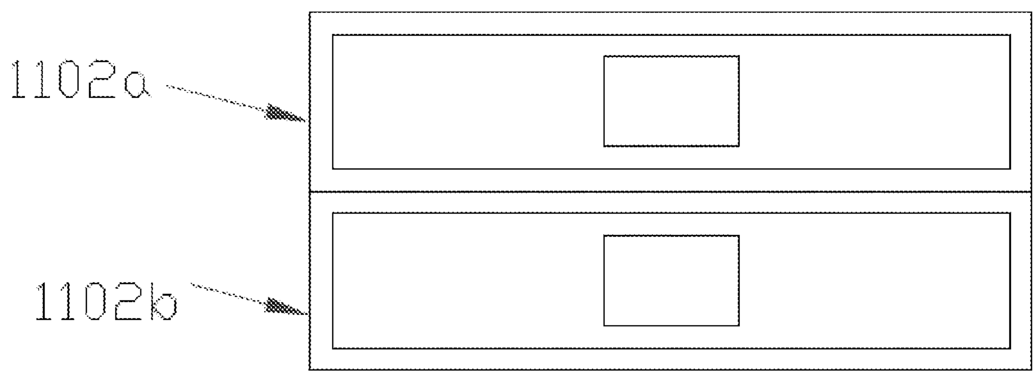


FIG. 47

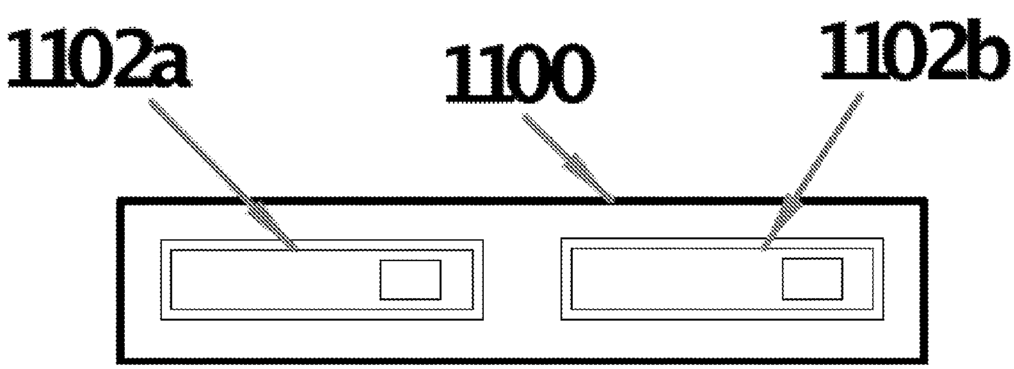


FIG. 48

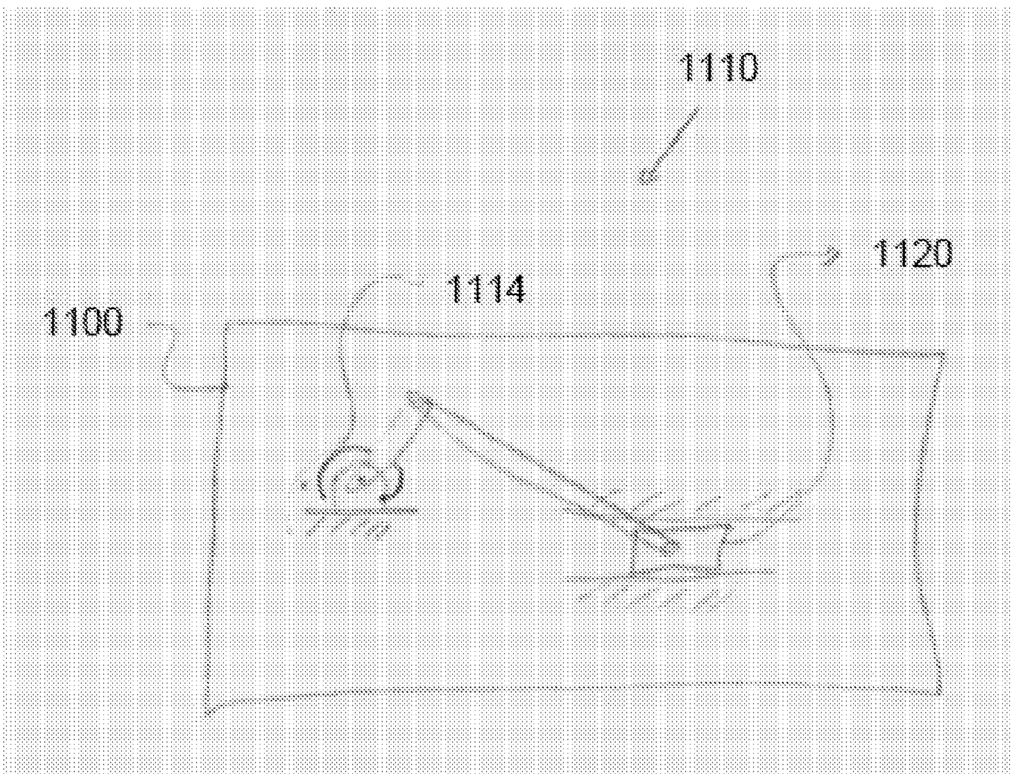


FIG. 49

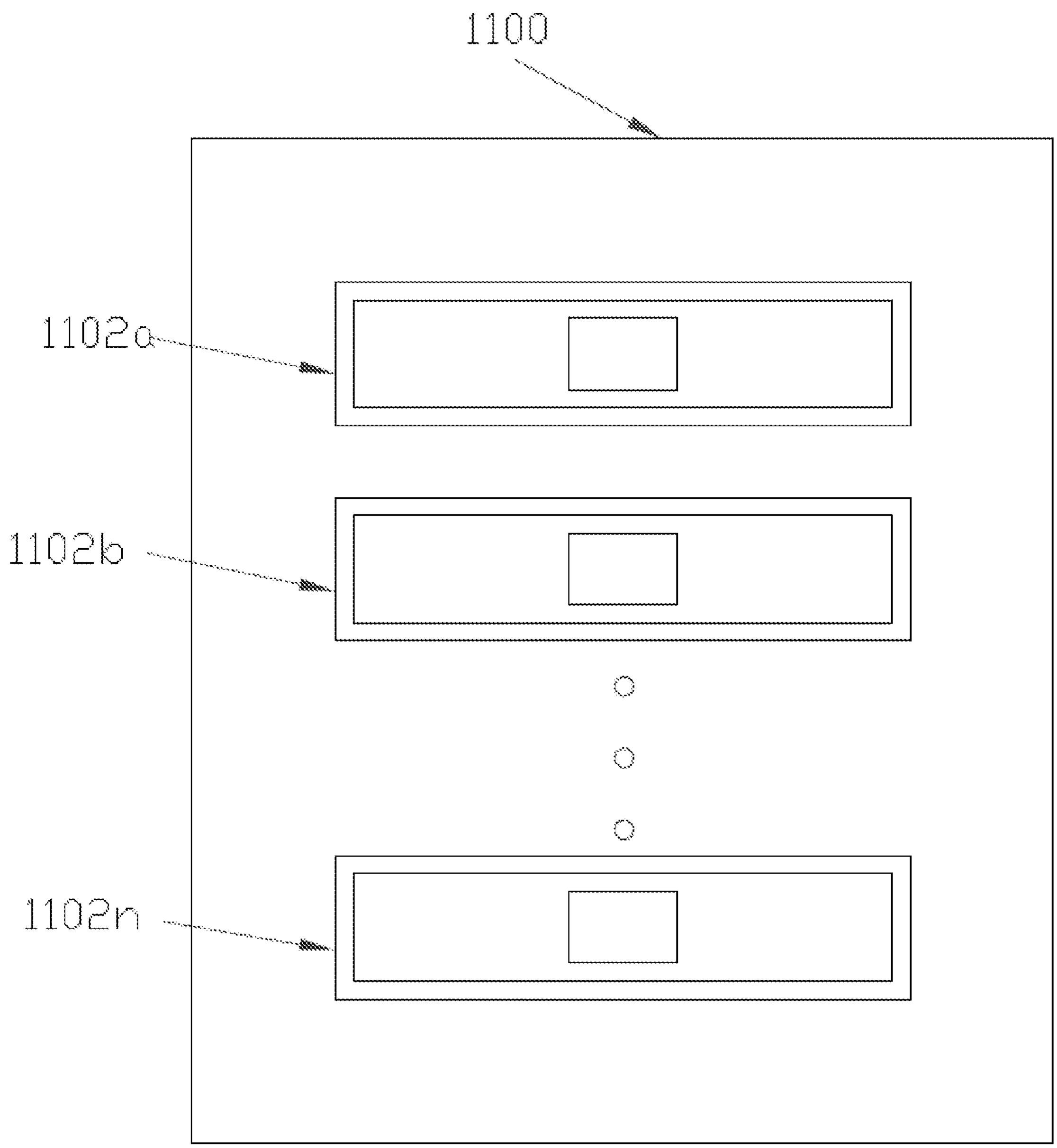


FIG. 50

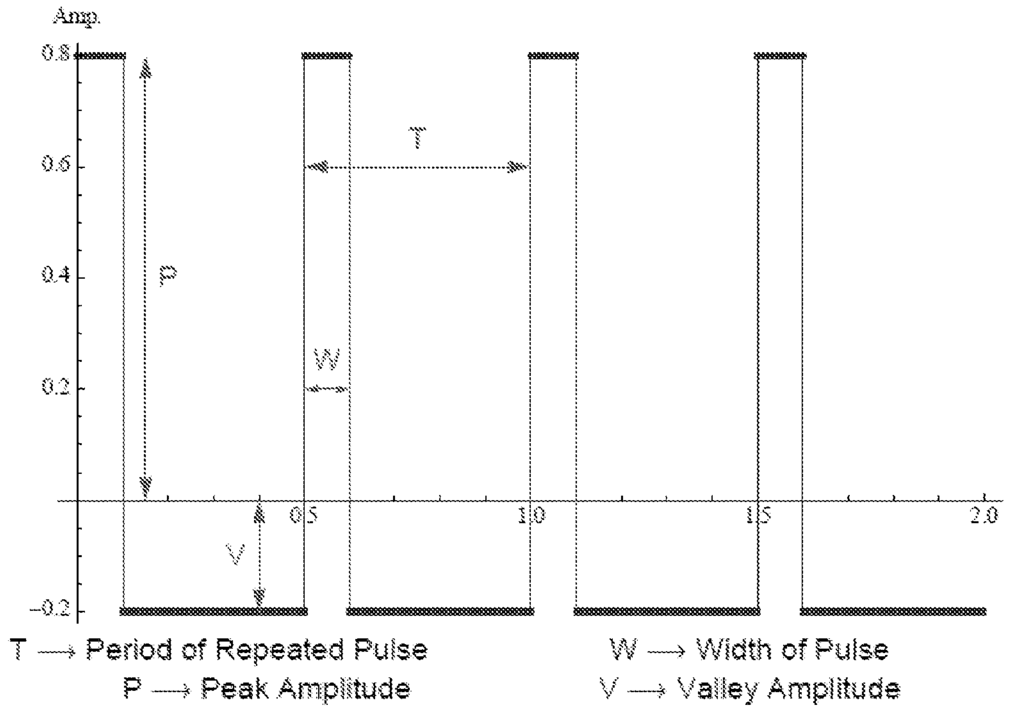

Amp.
0.8
0.6
0.4
0.2
-0.2
0.5
1.0
1.5
2.0
t
T
P
W
V
T ⟶ Period of Repeated Pulse
W ⟶ Width of Pulse
P ⟶ Peak Amplitude
V ⟶ Valley Amplitude

FIG. 51

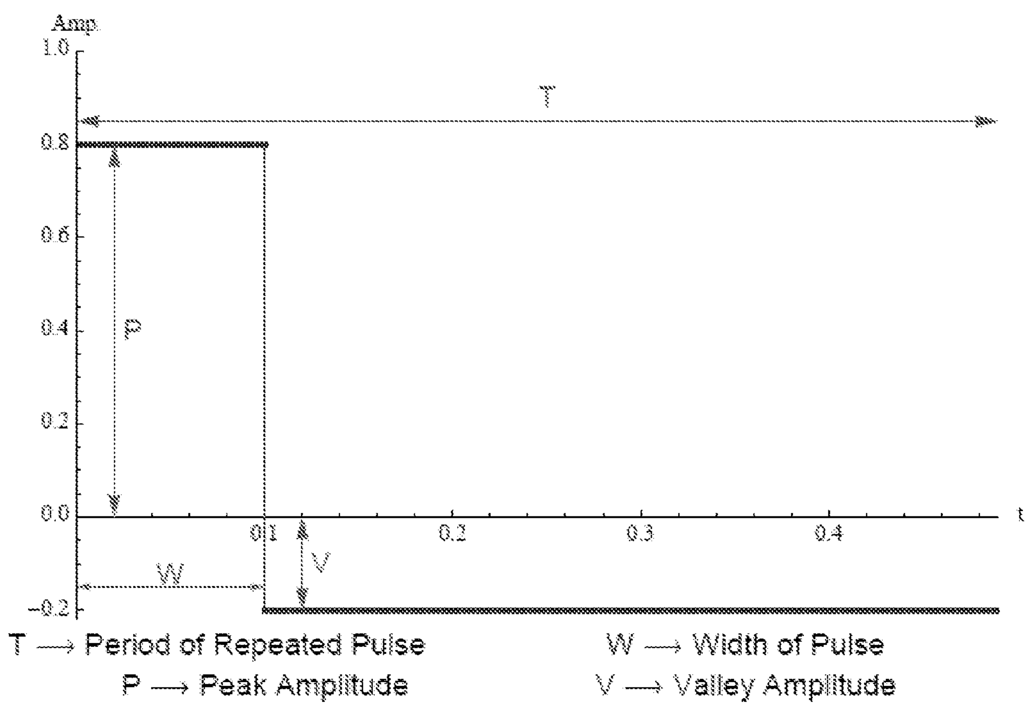

Amp.
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
0.1
0.2
0.3
0.4
t
T
P
W
V
T ⟶ Period of Repeated Pulse
W ⟶ Width of Pulse
P ⟶ Peak Amplitude
V ⟶ Valley Amplitude

FIG. 52

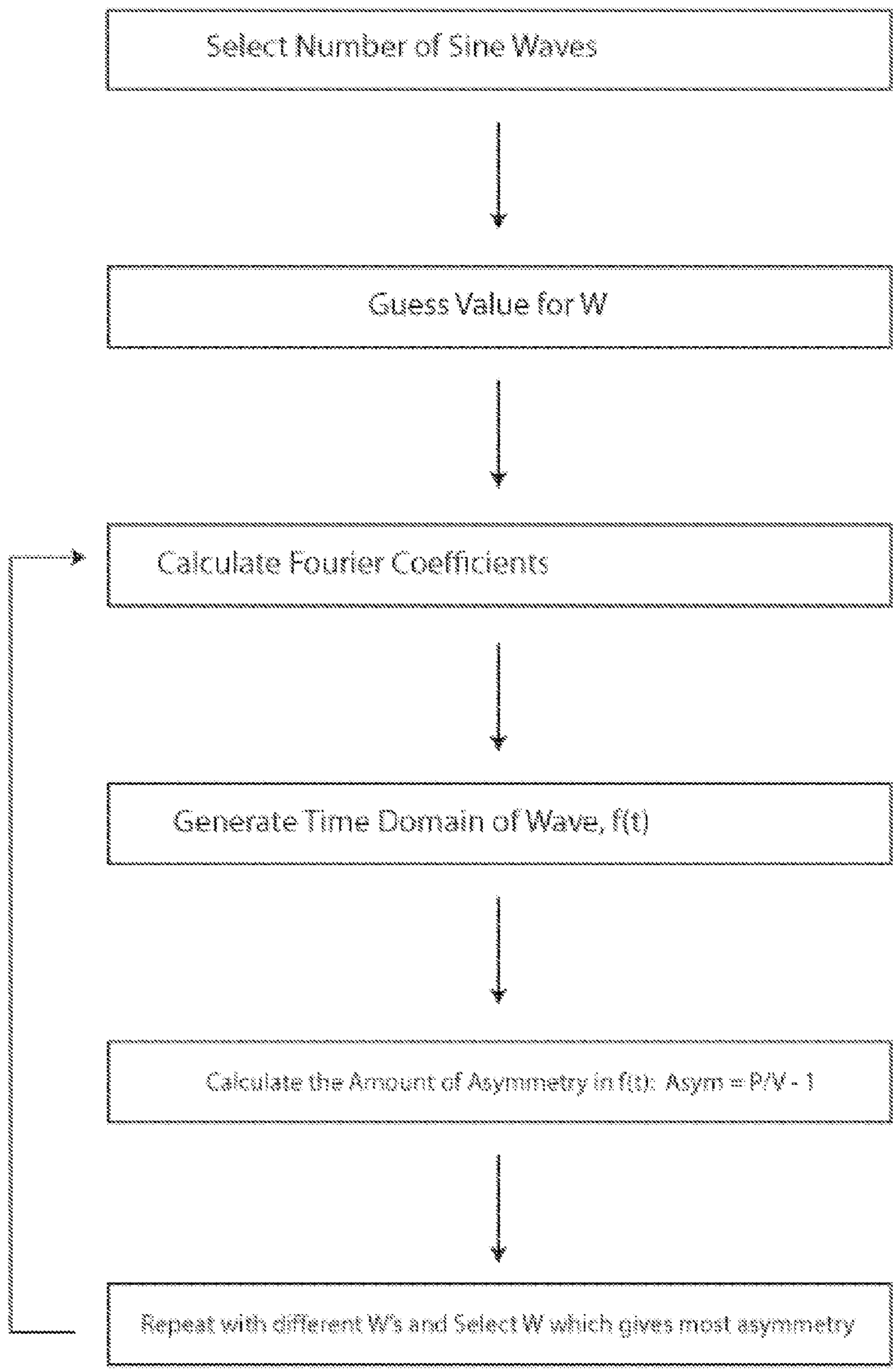

Select Number of Sine Waves
Guess Value for W
Calculate Fourier Coefficients
Generate Time Domain of Wave, f(t)
Calculate the Amount of Asymmetry in f(t): Asym = P/V - 1
Repeat with different W's and Select W which gives most asymmetry

FIG. 53

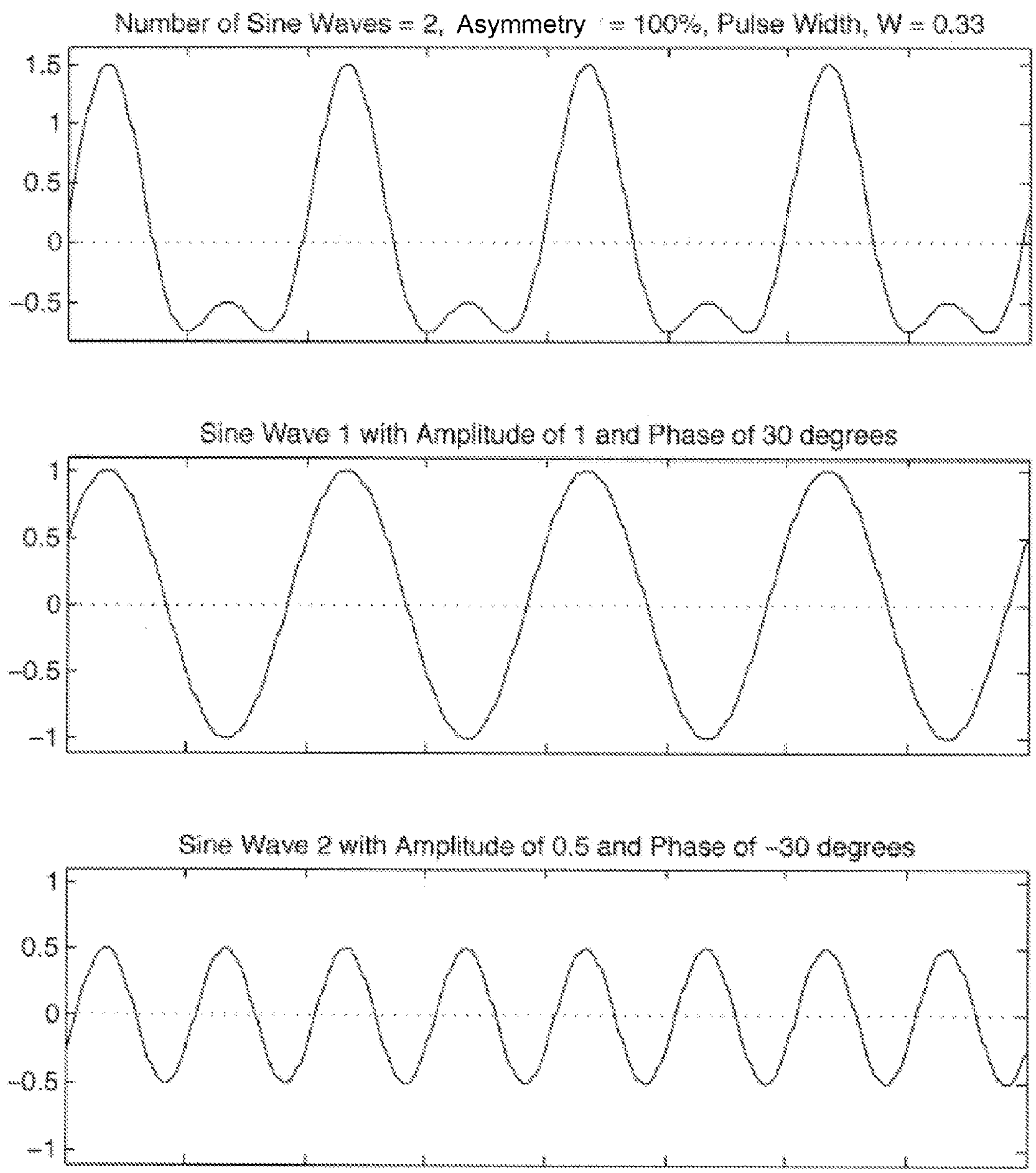

Number of Sine Waves = 2, Asymmetry = 100%, Pulse Width, W = 0.33
1.5
1
0.5
0
−0.5
Sine Wave 1 with Amplitude of 1 and Phase of 30 degrees
1
0.5
0
−0.5
−1
Sine Wave 2 with Amplitude of 0.5 and Phase of −30 degrees
1
0.5
0
−0.5
−1

FIG. 54

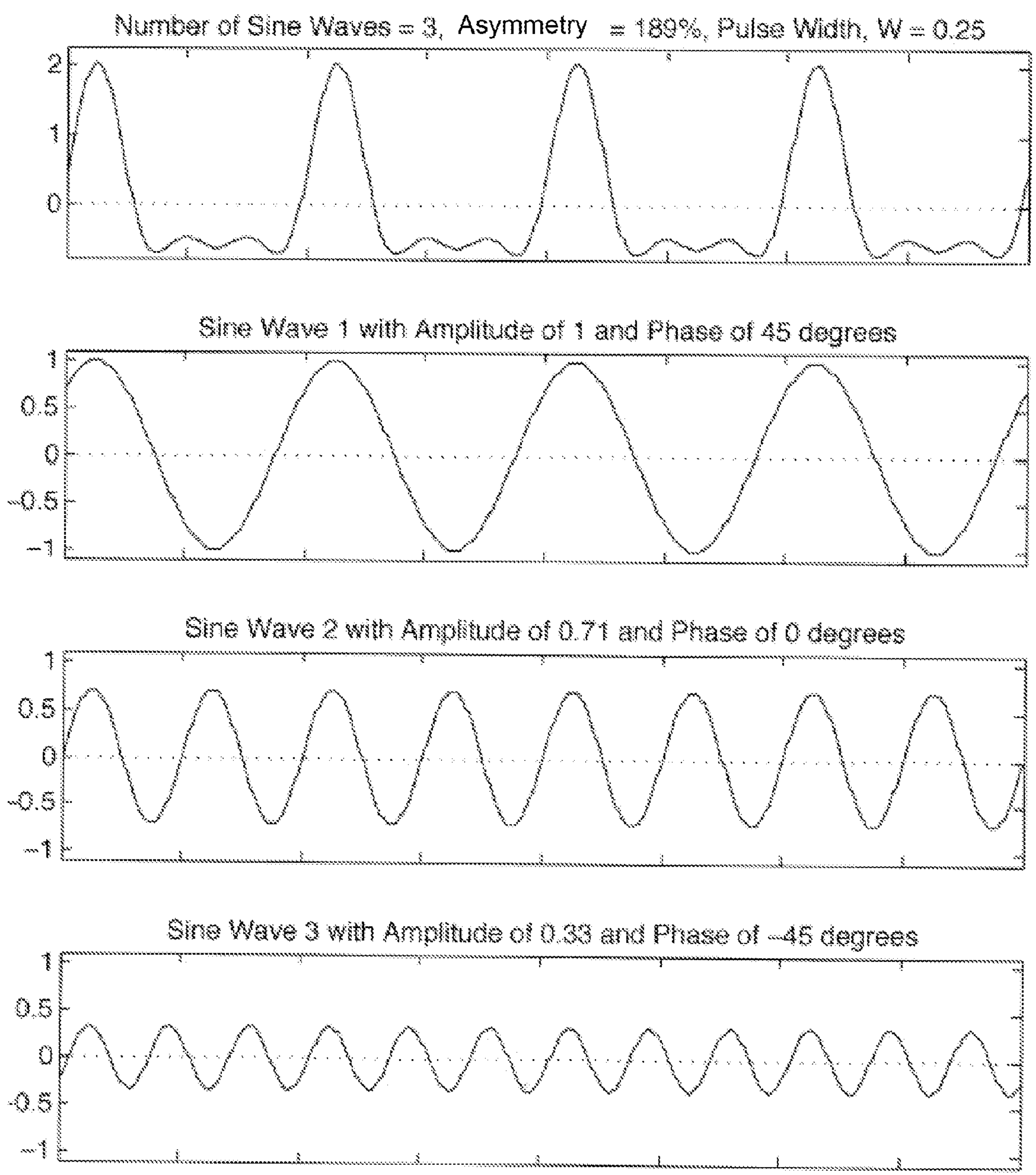

Number of Sine Waves = 3, Asymmetry = 189%, Pulse Width, W = 0.25
2
1
0
Sine Wave 1 with Amplitude of 1 and Phase of 45 degrees
1
0.5
0
−0.5
−1
Sine Wave 2 with Amplitude of 0.71 and Phase of 0 degrees
1
0.5
0
−0.5
−1
Sine Wave 3 with Amplitude of 0.33 and Phase of −45 degrees
1
0.5
0
−0.5
−1

FIG. 55

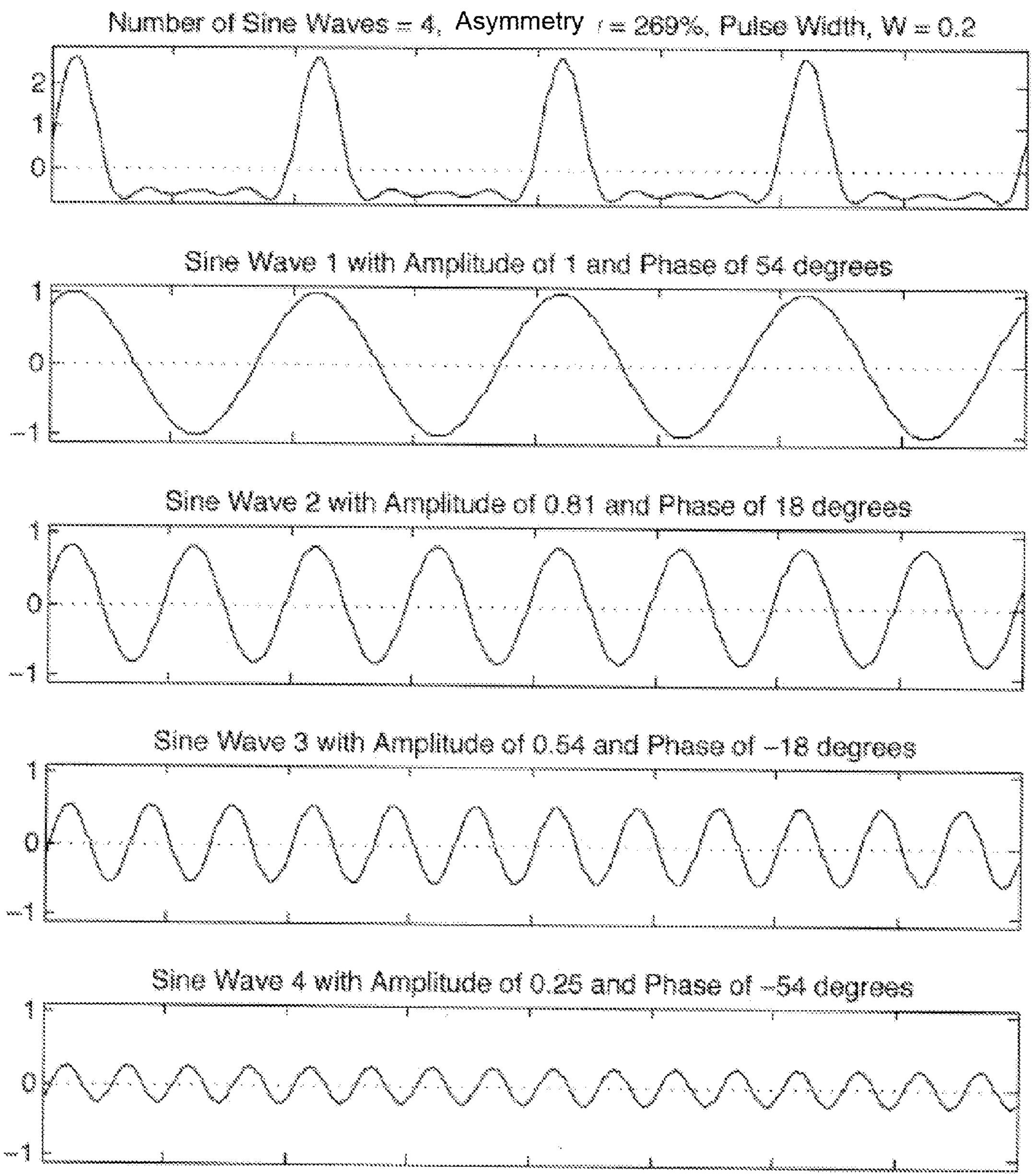

Number of Sine Waves = 4, Asymmetry r = 269%, Pulse Width, W = 0.2
2
1
0
Sine Wave 1 with Amplitude of 1 and Phase of 54 degrees
1
0
−1
Sine Wave 2 with Amplitude of 0.81 and Phase of 18 degrees
1
0
−1
Sine Wave 3 with Amplitude of 0.54 and Phase of −18 degrees
1
0
−1
Sine Wave 4 with Amplitude of 0.25 and Phase of −54 degrees
1
0
−1

FIG. 56

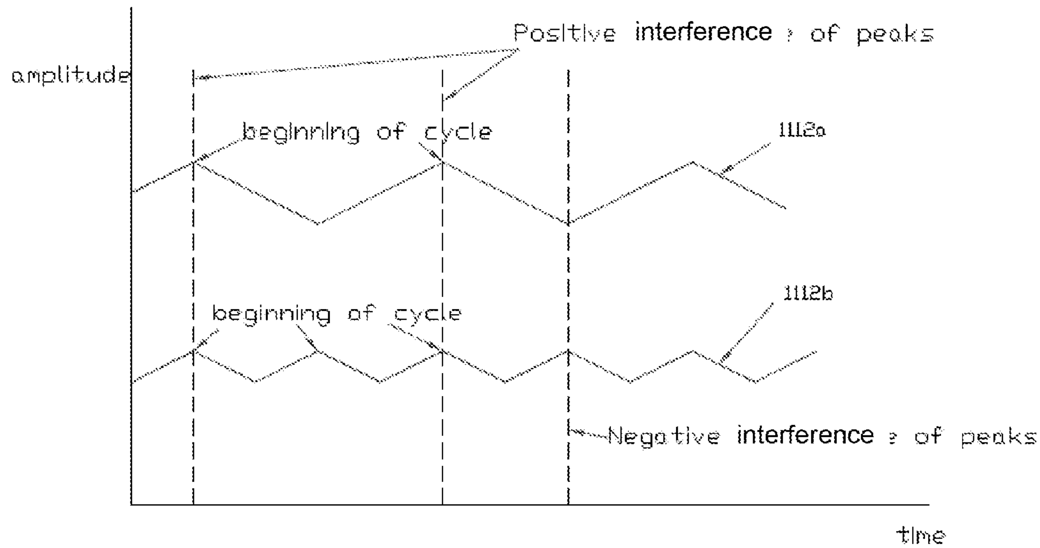

Positive interference ? of peaks
amplitude
beginning of cycle
1112a
beginning of cycle
1112b
Negative interference ? of peaks
time

FIG. 57

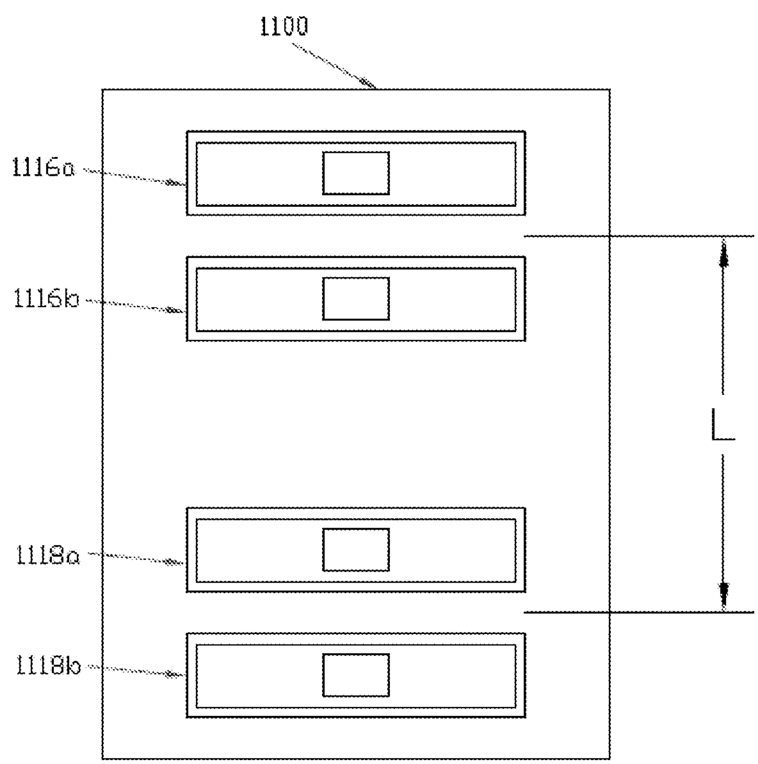

1100
1116a
1116b
L
1118a
1118b

FIG. 58

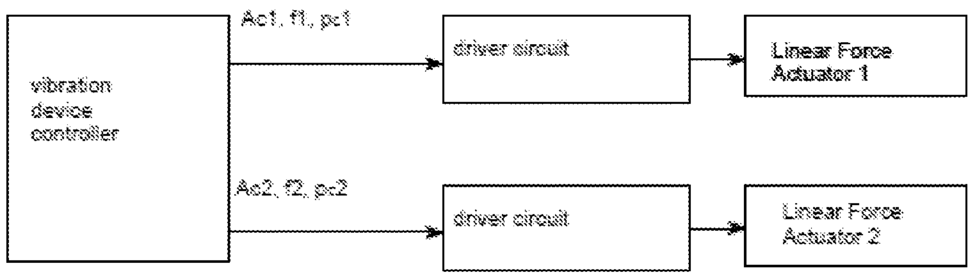


FIG. 59

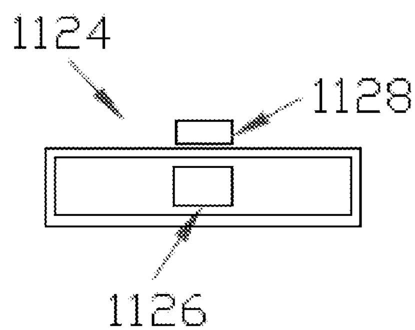


FIG. 60

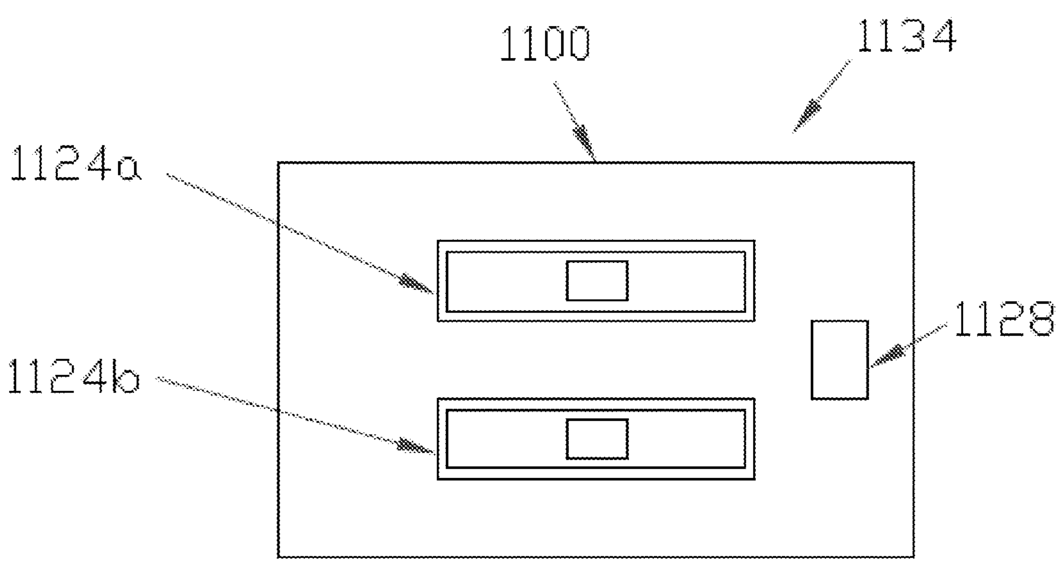
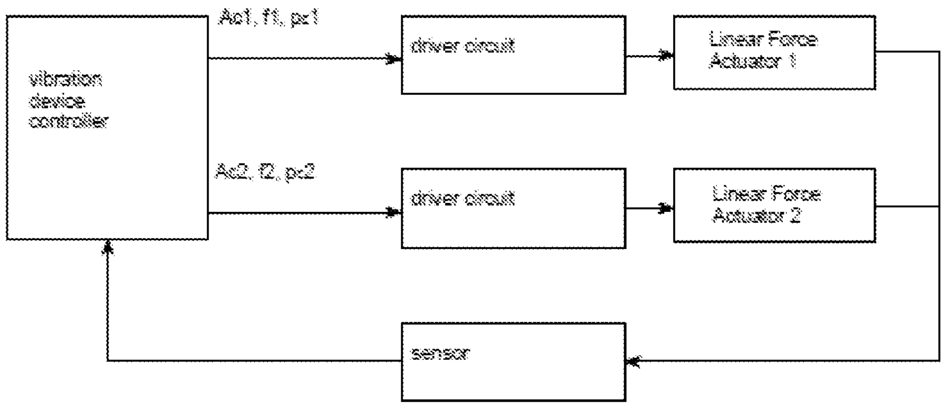
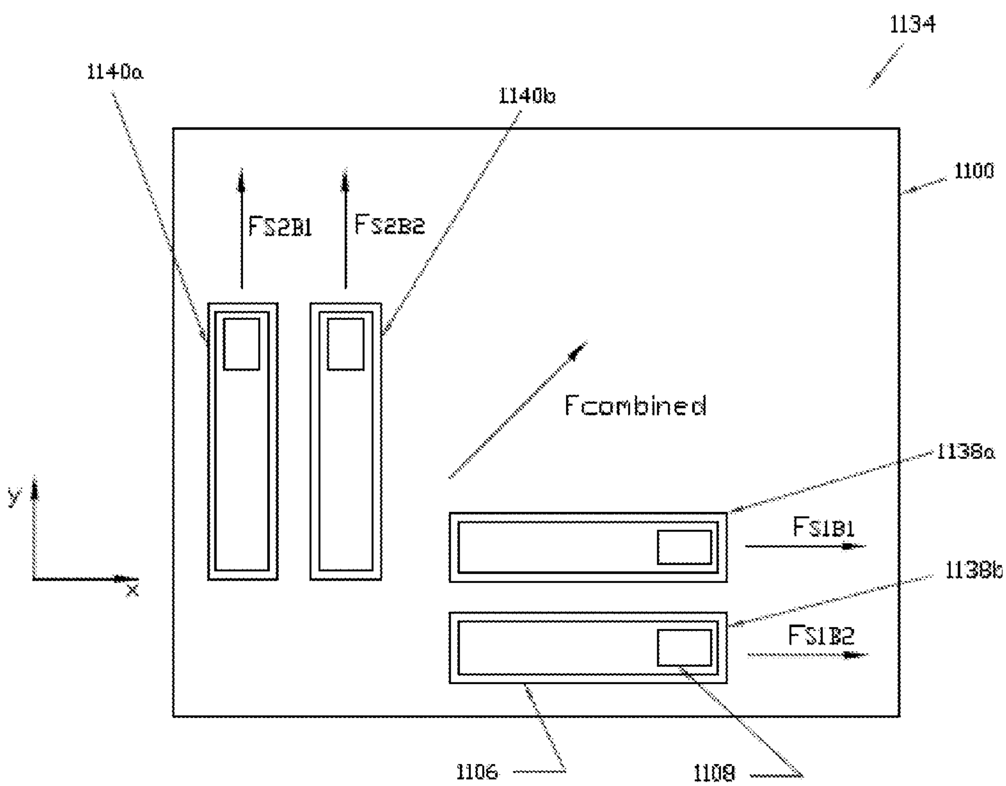


FIG. 61



Ac1, f1, pc1
vibration device controller
driver circuit
Linear Force Actuator 1
Ac2, f2, pc2
driver circuit
Linear Force Actuator 2
sensor

FIG. 62



1134
1140a
1140b
1100
FS2B1
FS2B2
Fcombined
1138a
FS1B1
1138b
FS1B2
y
x
1106
1108

FIG. 63

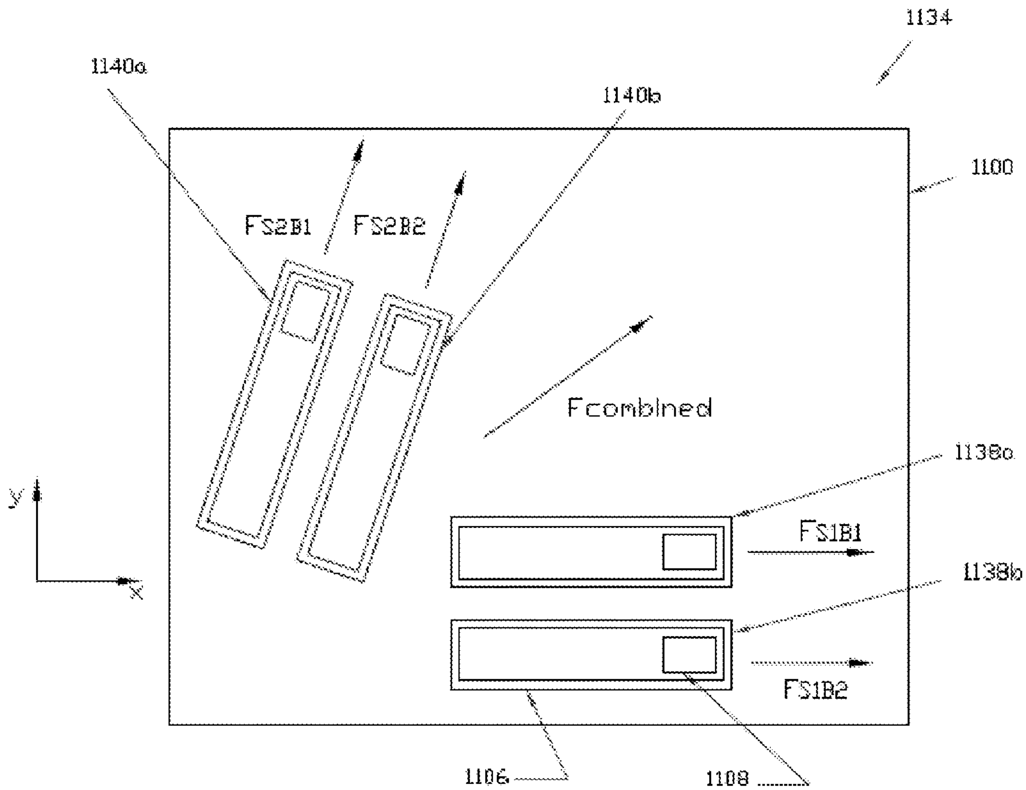

1134
1140a
1140b
1100
FS2B1
FS2B2
Fcombined
1138a
FS1B1
1138b
FS1B2
y
x
1106
1108

FIG. 64

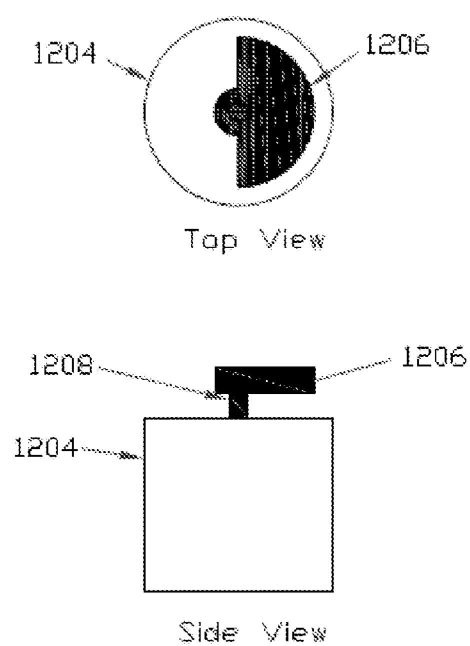

1204
1206
Top View
1208
1206
1204
Side View

FIG. 65

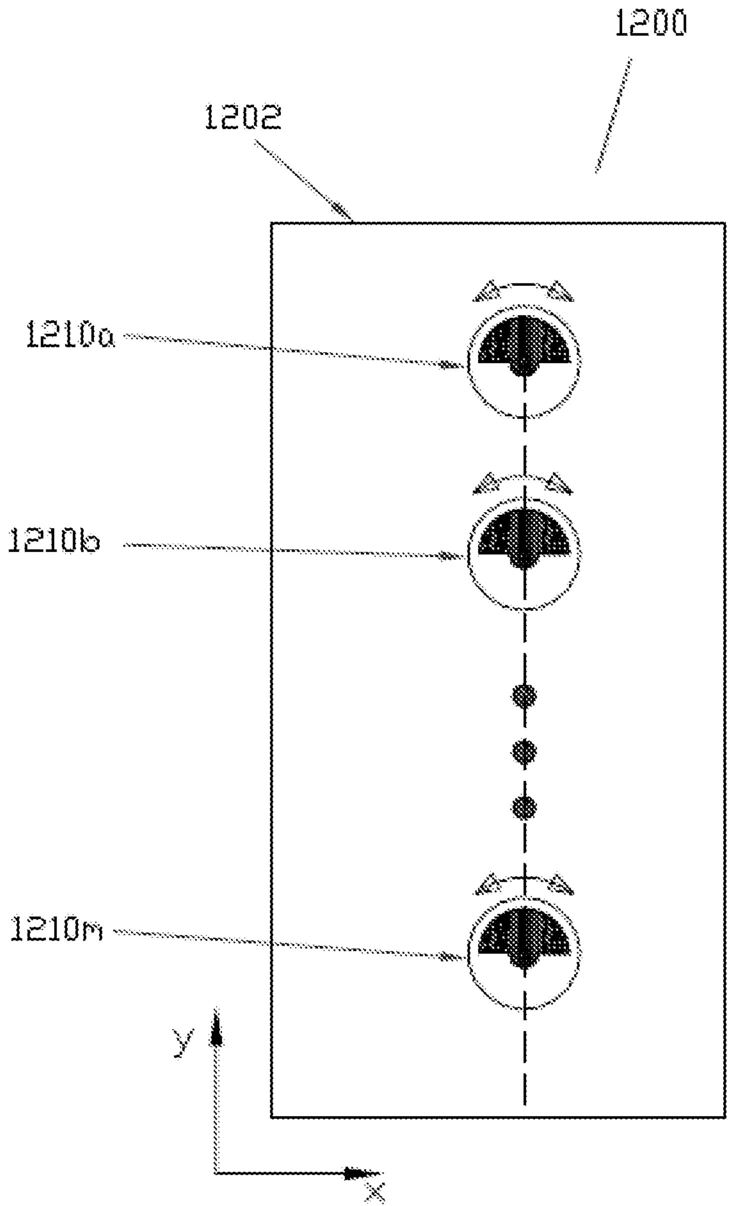


FIG. 66

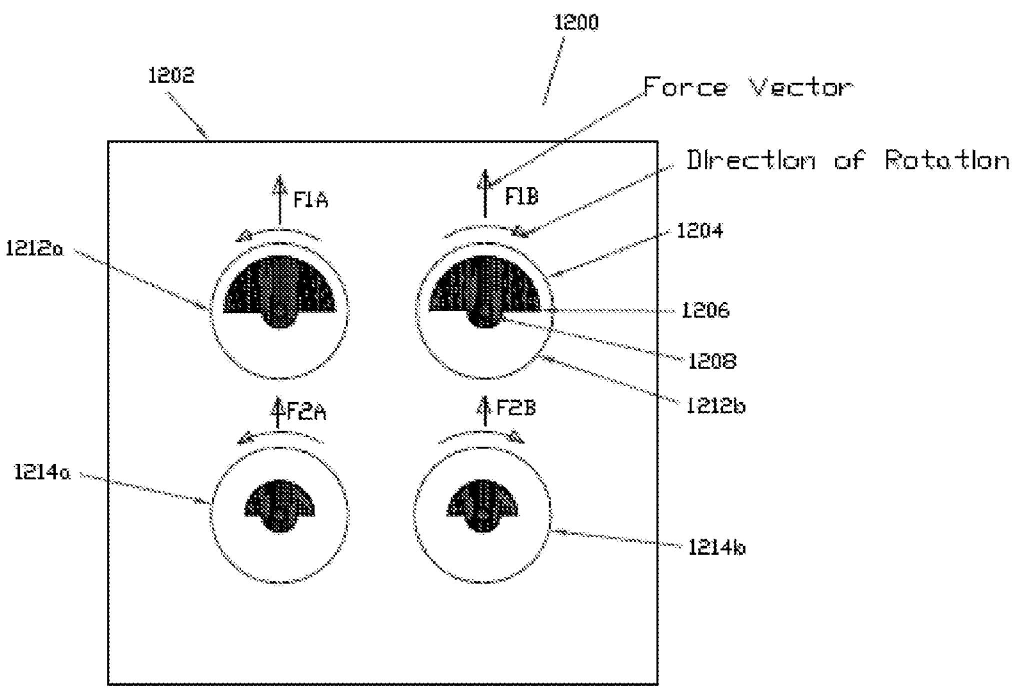

1200
1202
Force Vector
Direction of Rotation
F1A
F1B
1204
1212a
1206
1208
1212b
F2A
F2B
1214a
1214b

FIG. 67

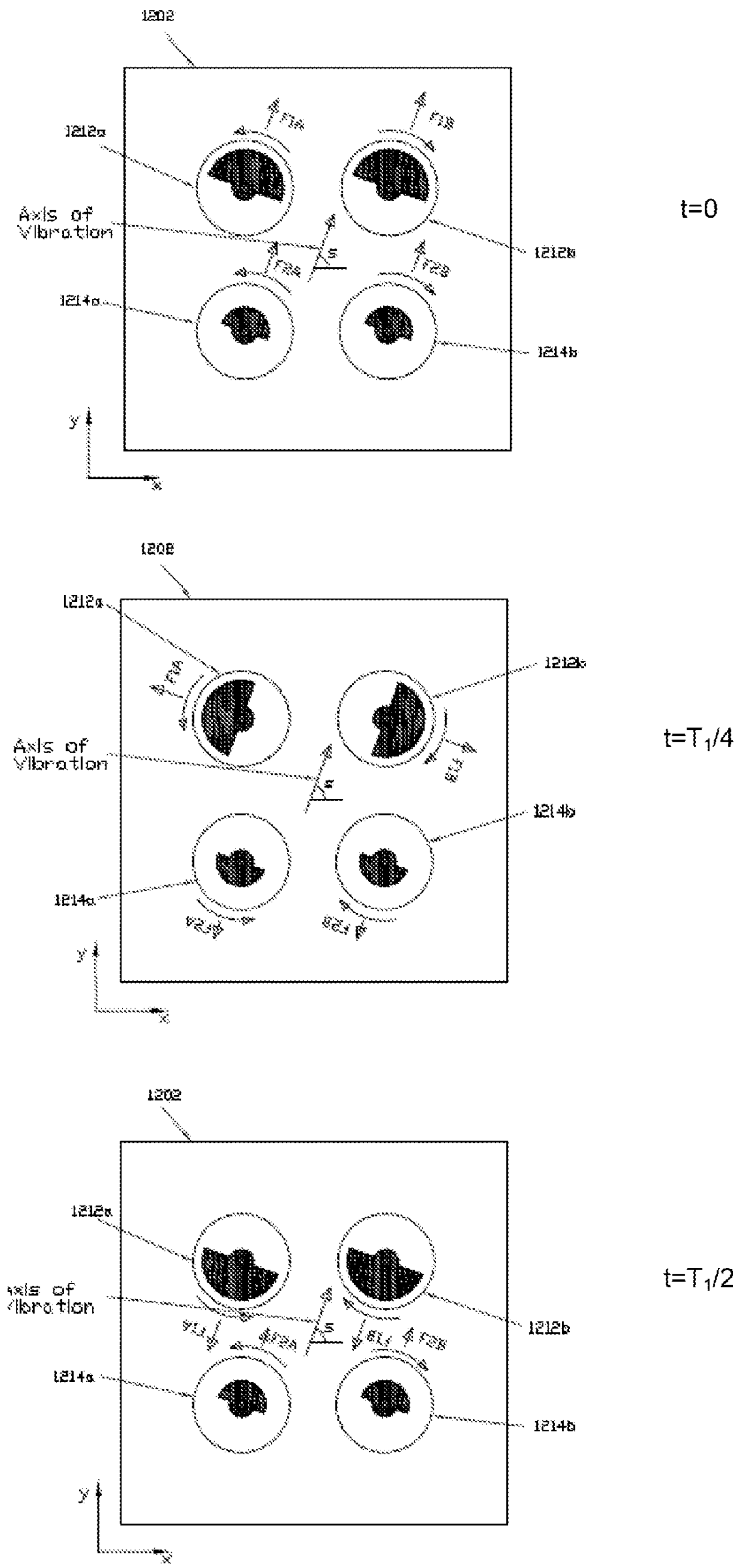

1202
1212a
F1A
F1B
Axis of Vibration
s
1212b
F2A
F2B
1214a
1214b
y
x
t=0
t=T1/4
t=T1/2

FIG. 68

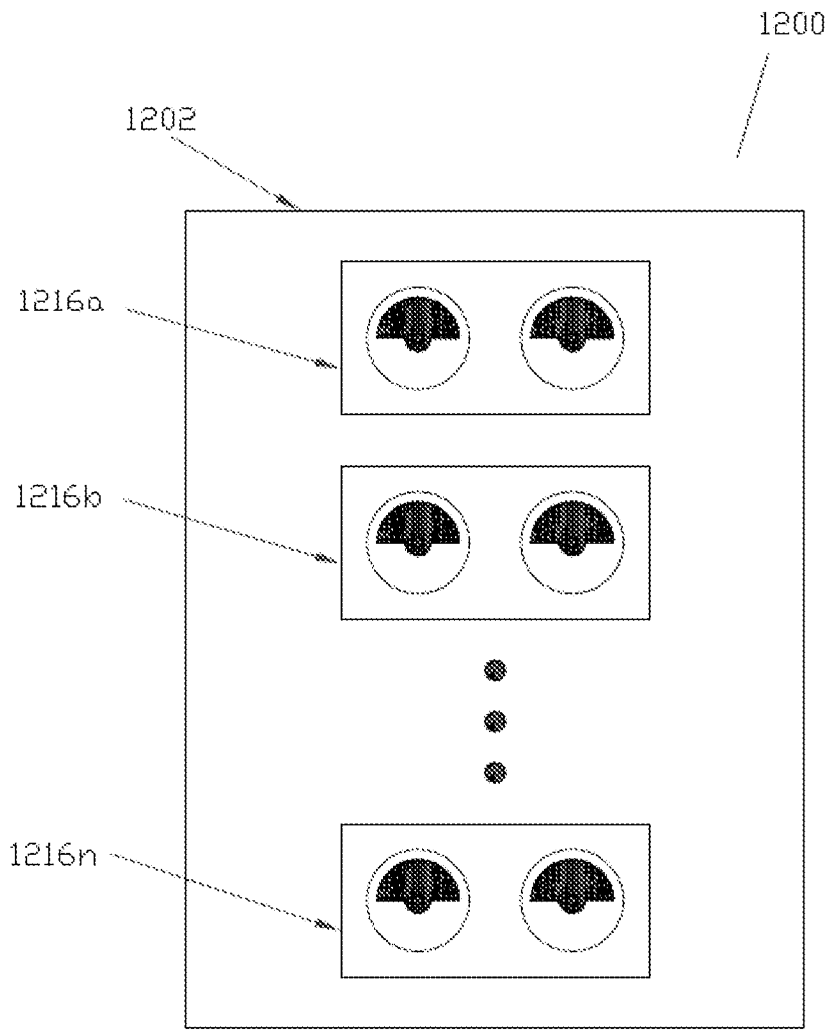


FIG. 69

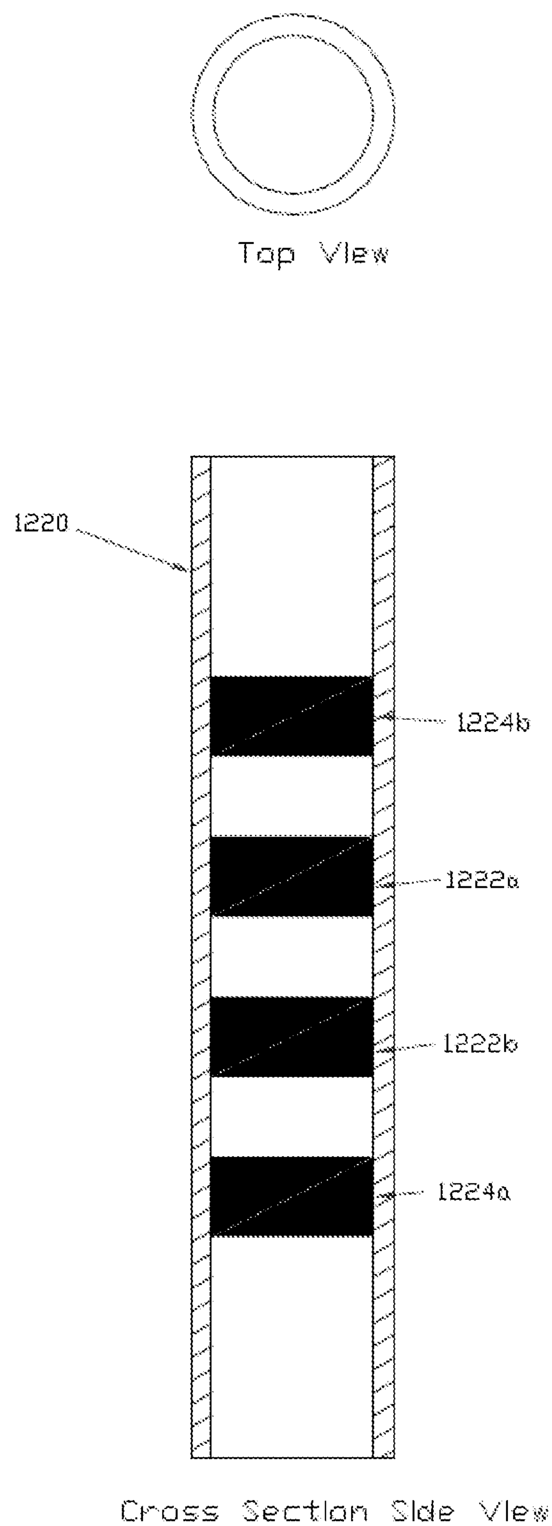

Top View
1220
1224b
1222a
1222b
1224a
Cross Section Side View

FIG. 70

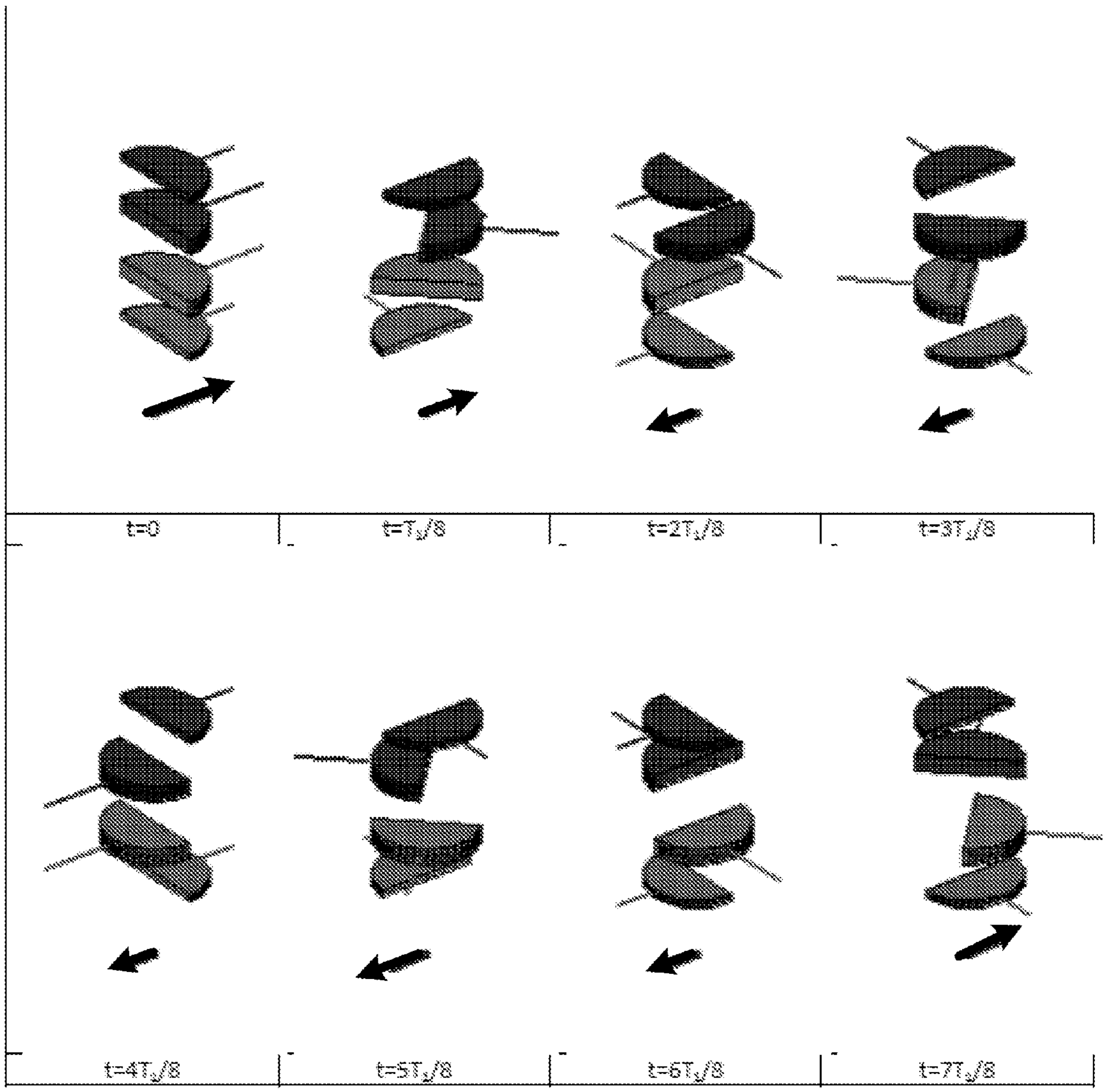


FIG. 71

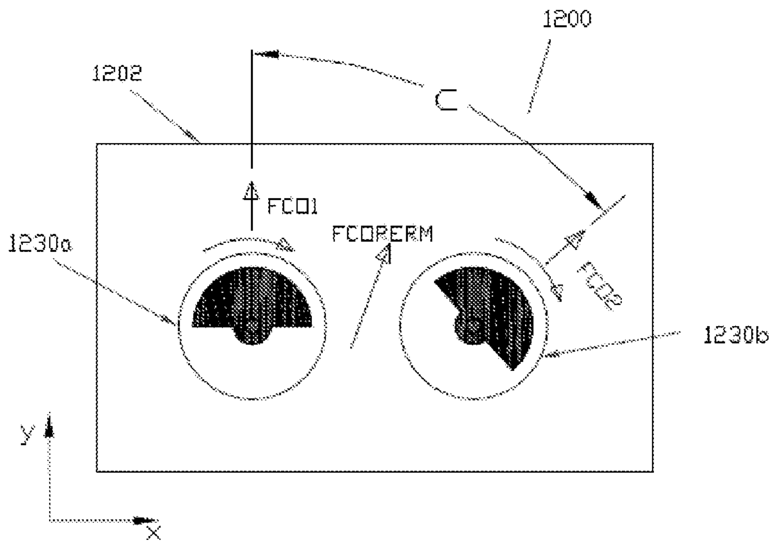

1200
1202
C
FCO1
FCOPERM
FCO2
1230a
1230b
y
x

FIG. 72

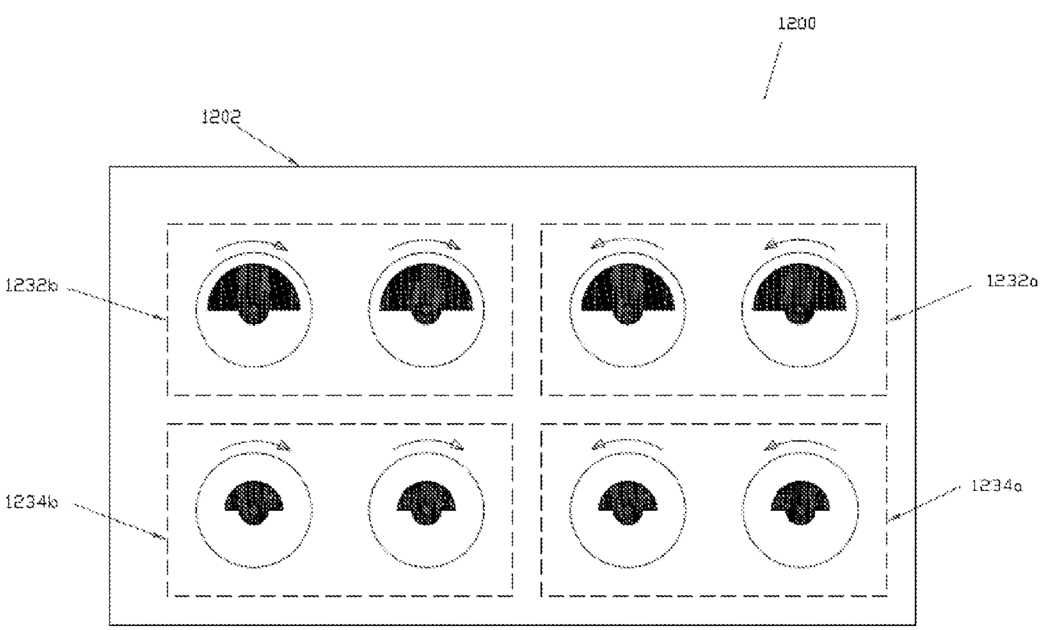
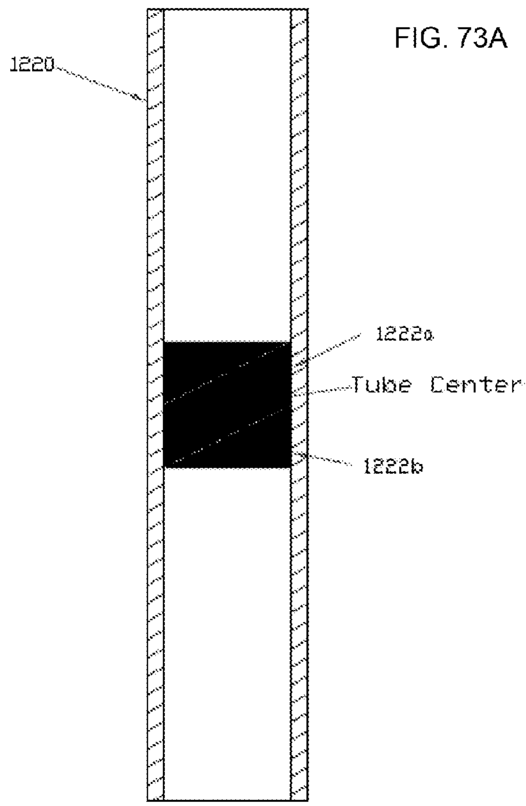
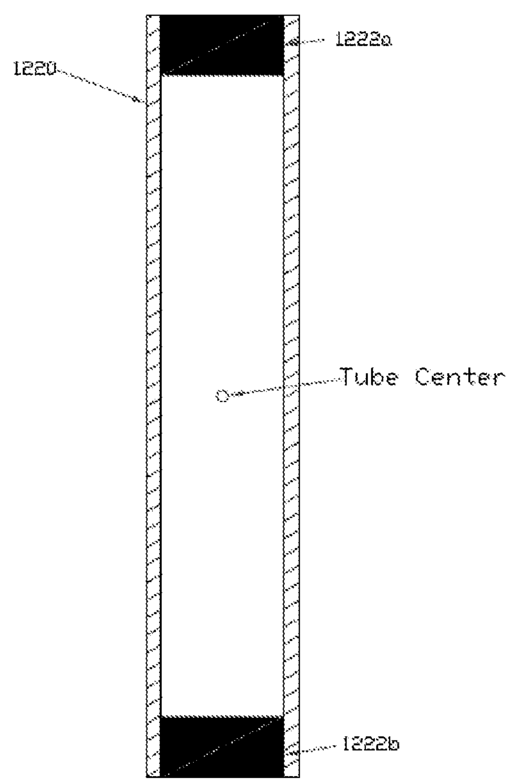

1200
1202
1232b
1232a
1234b
1234a

FIG. 73A



Cross Section Side View

FIG. 73B



Cross Section Side View

FIG. 74

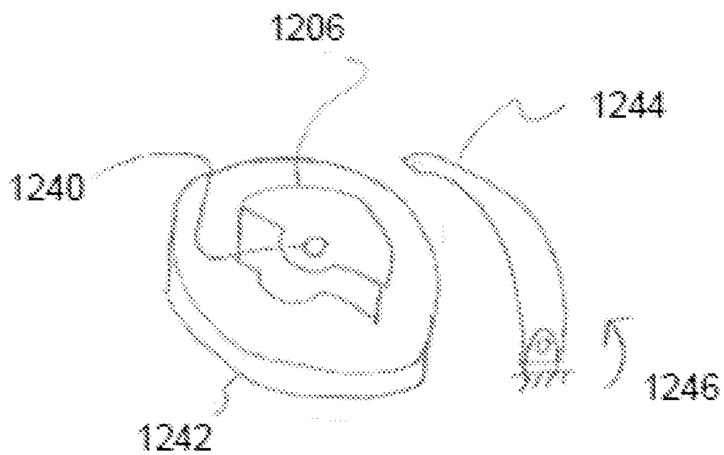


FIG. 75

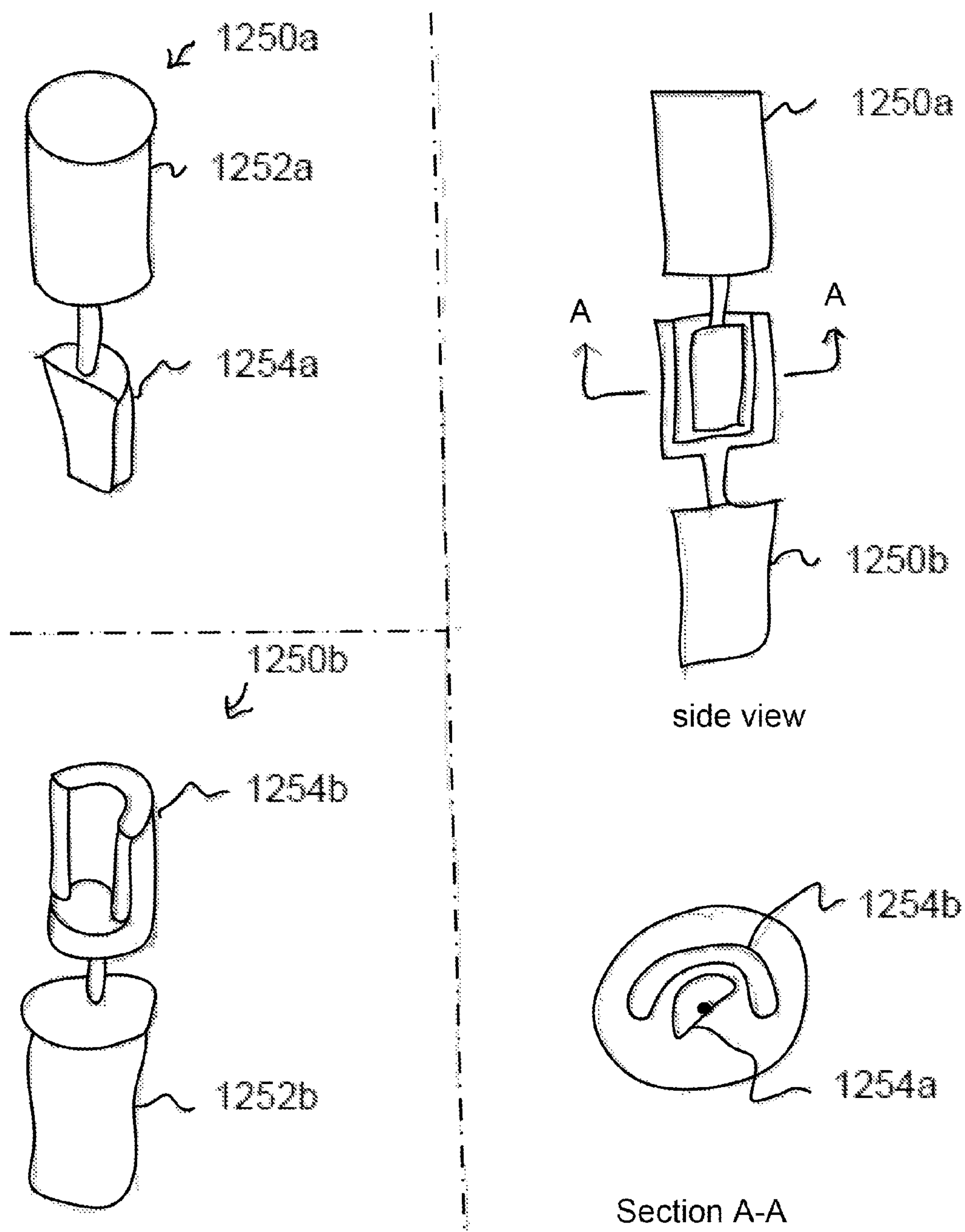
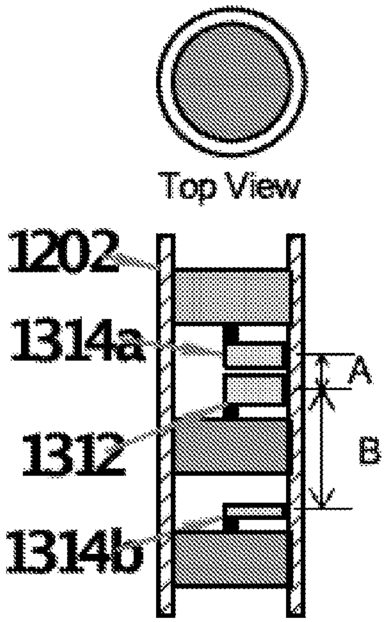
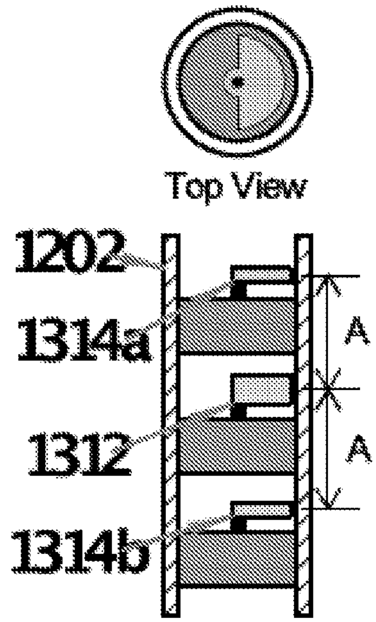

1250a
1252a
1254a
1250b
1254b
1252b
1250a
A
A
1250b
side view
1254b
1254a
Section A-A

FIG. 76A



Cross Section Side View

FIG. 76B



Cross Section Side View

FIG. 77

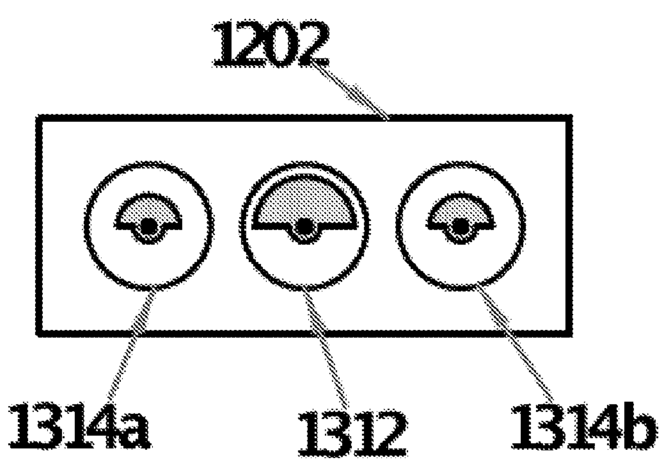


FIG. 78

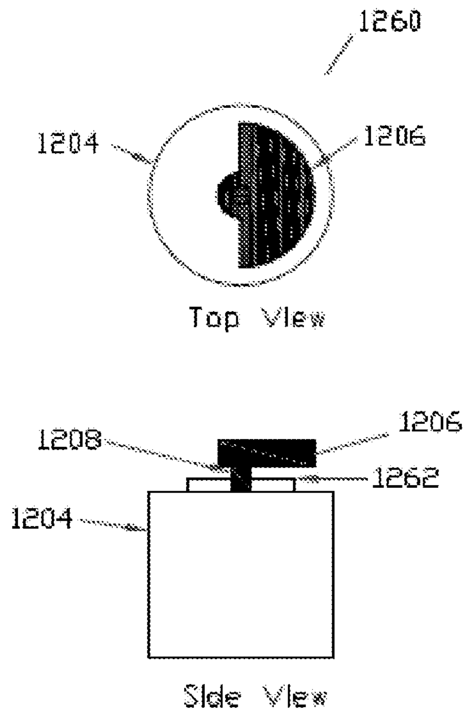

1260
1204
1206
Top View
1208
1206
1262
1204
Side View

FIG. 79

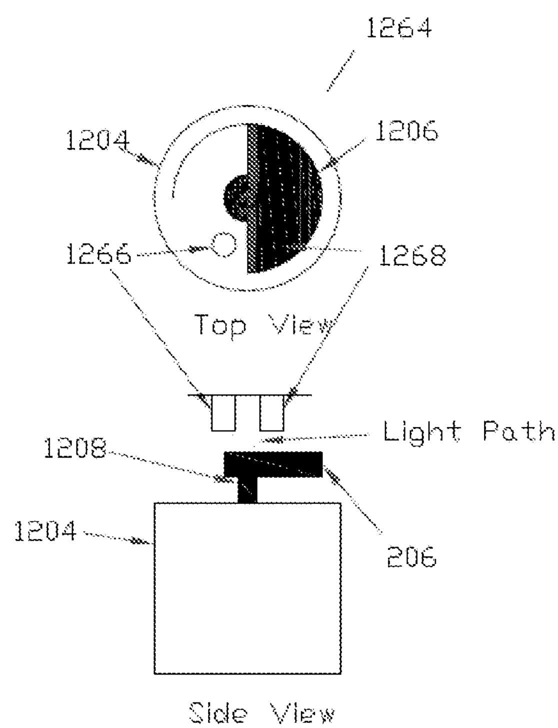

1264
1204
1206
1266
1268
Top View
Light Path
1208
1204
206
Side View

FIG. 80

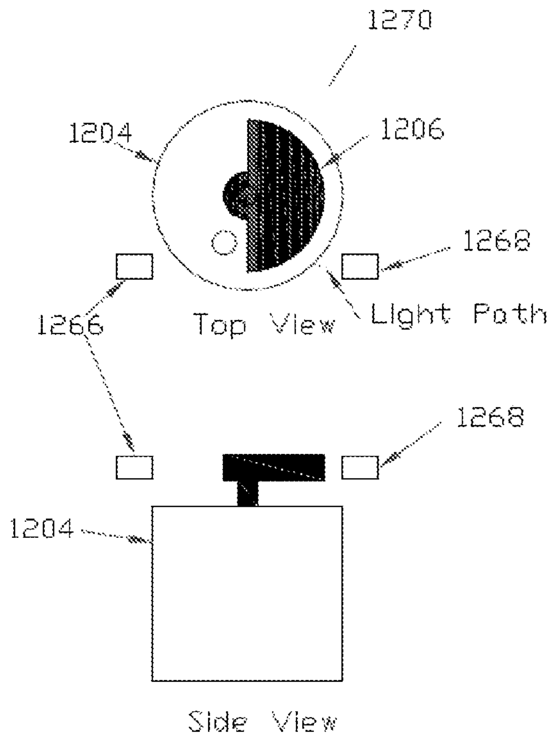

1270
1204
1206
1268
1266
Top View
Light Path
1268
1204
Side View

FIG. 81

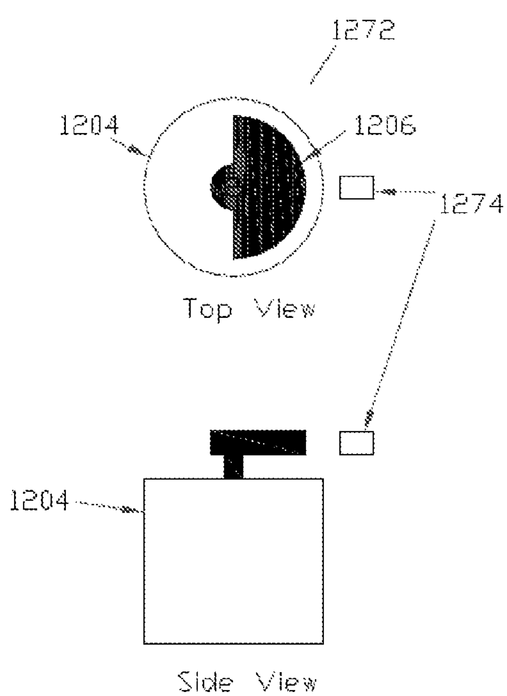

1272
1204
1206
1274
Top View
1204
Side View

FIG. 82

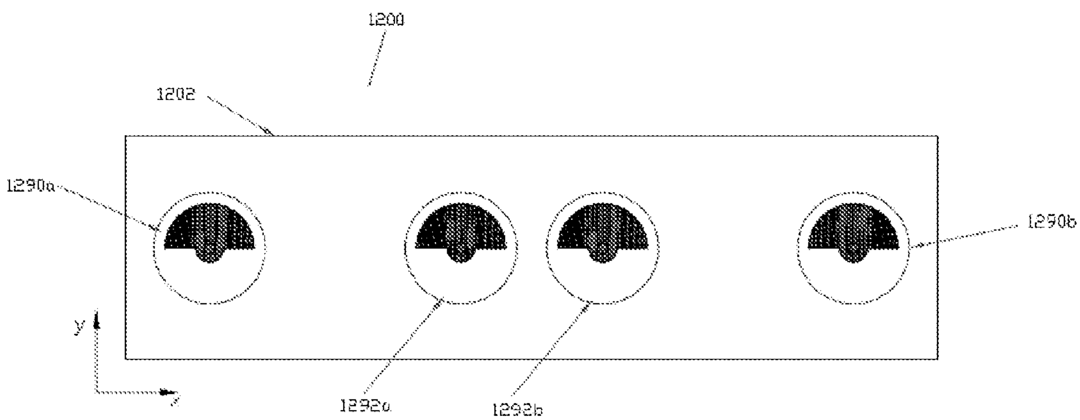


FIG. 83

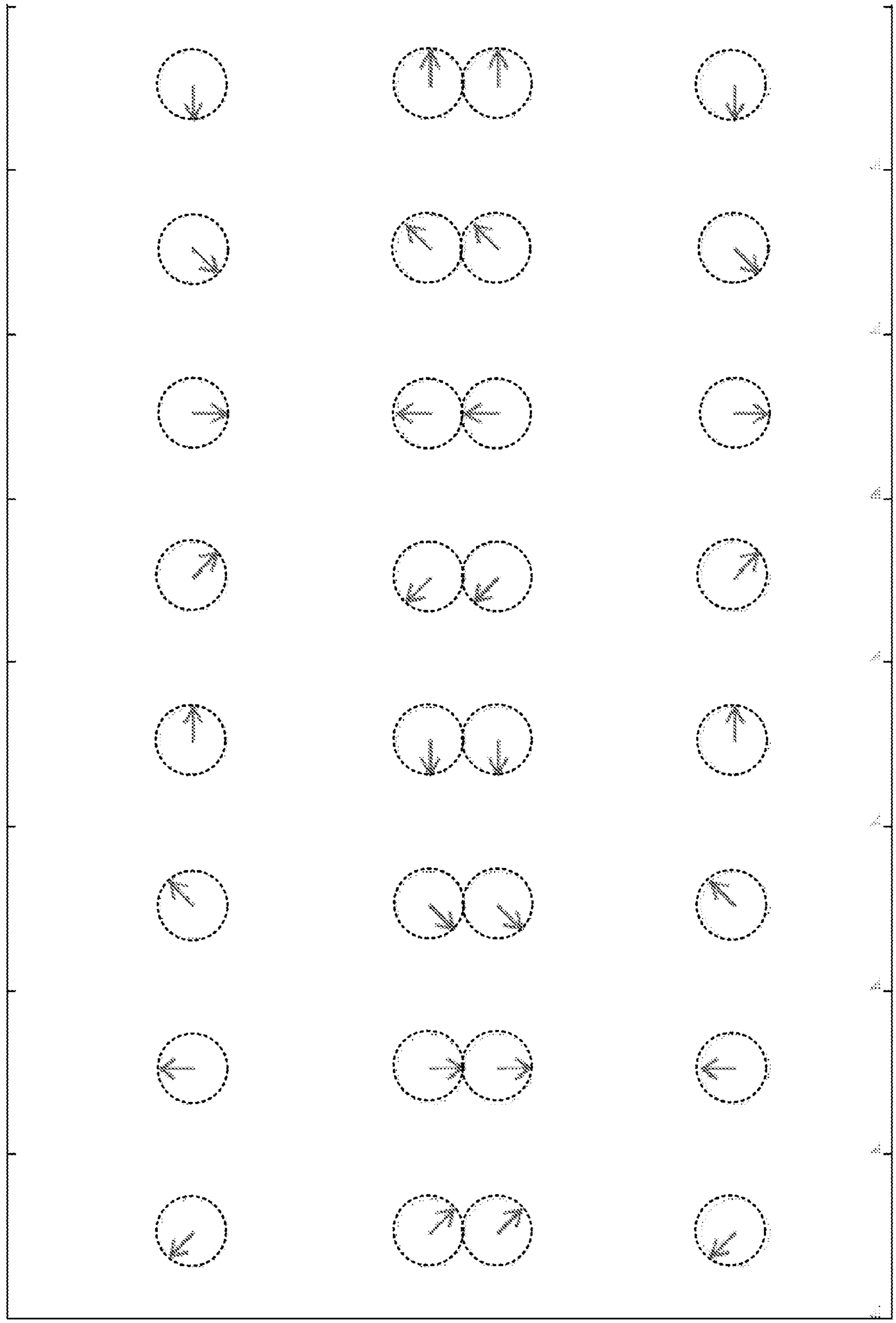


FIG. 84

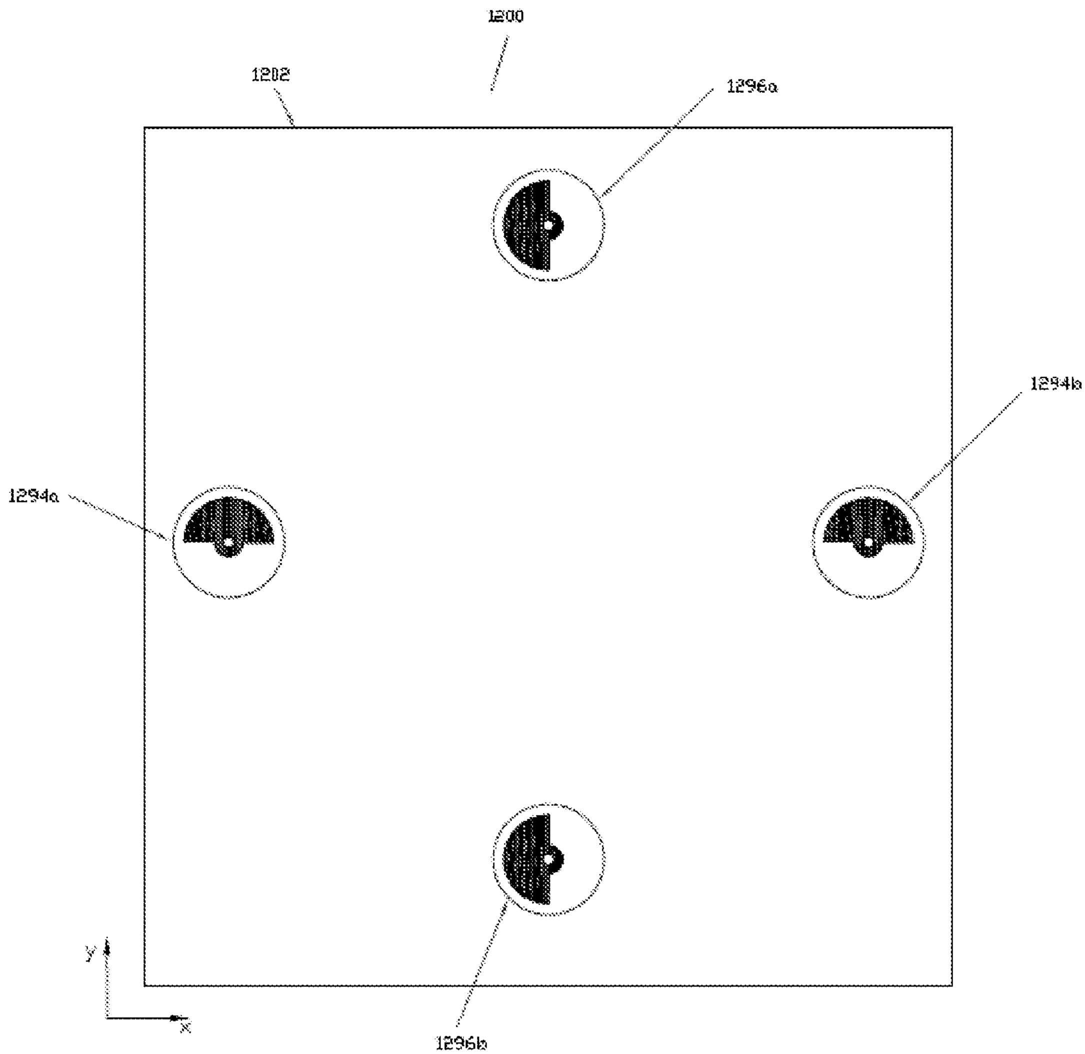


FIG. 85A

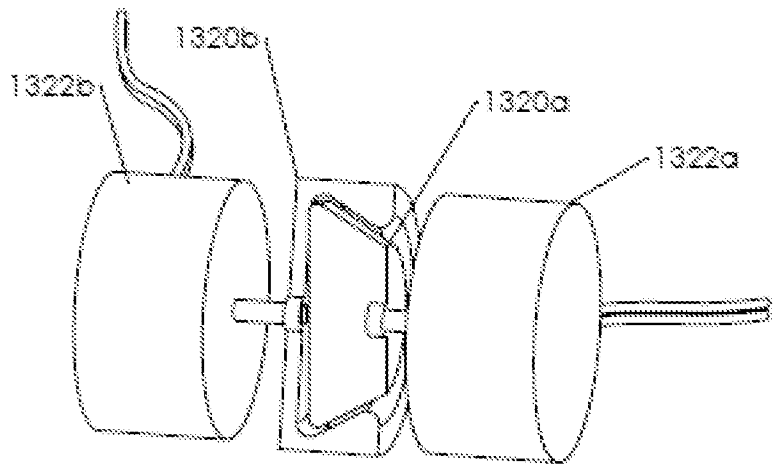


FIG. 85B

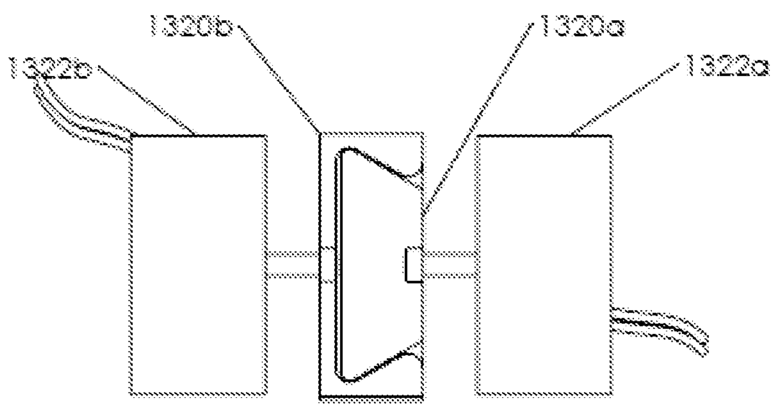


FIG. 86A

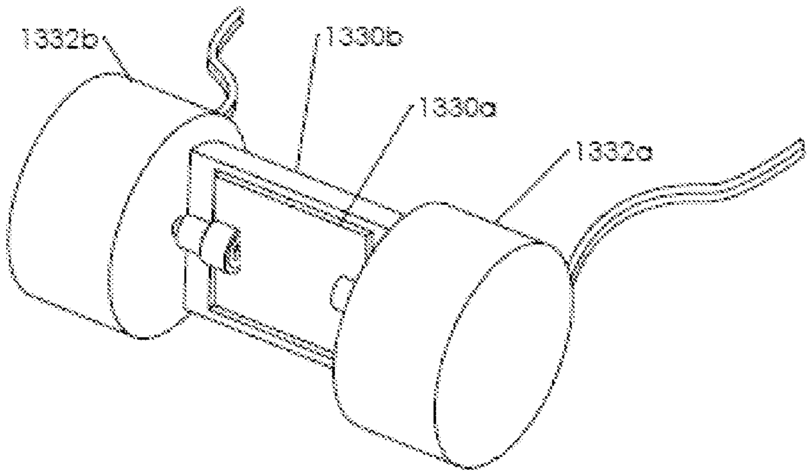


FIG. 86B

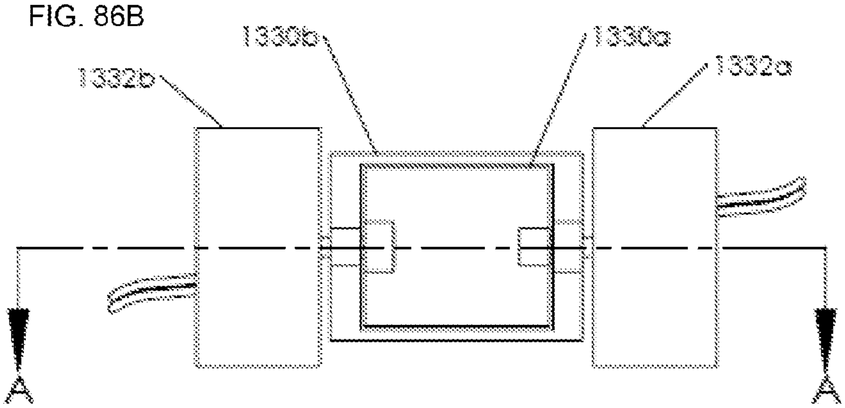
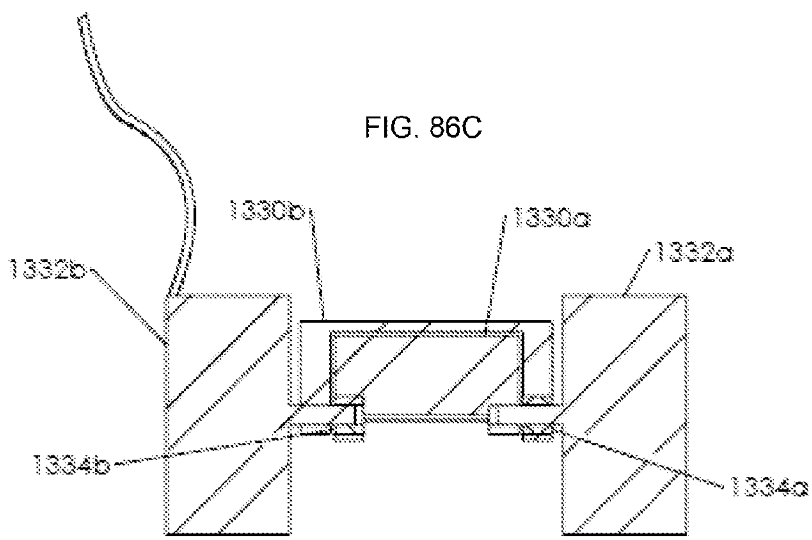


FIG. 86C



SECTION A-A

FIG. 87

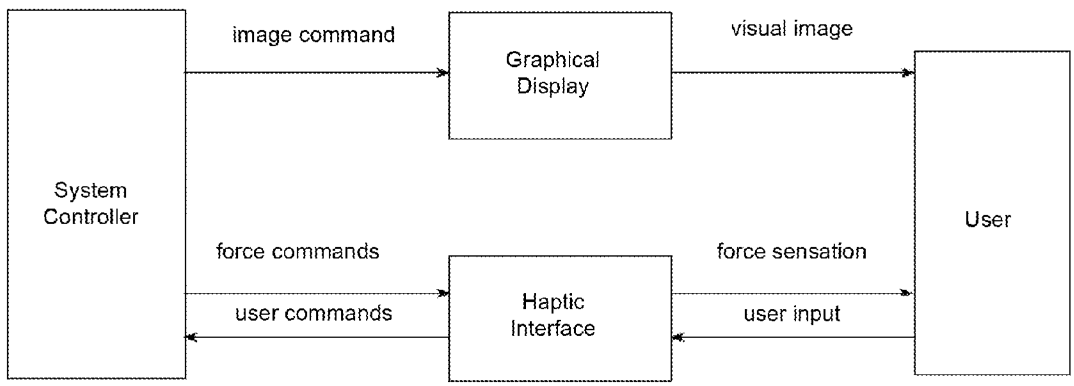

System Controller
image command
Graphical Display
visual image
User
force commands
user commands
Haptic Interface
force sensation
user input

FIG. 88

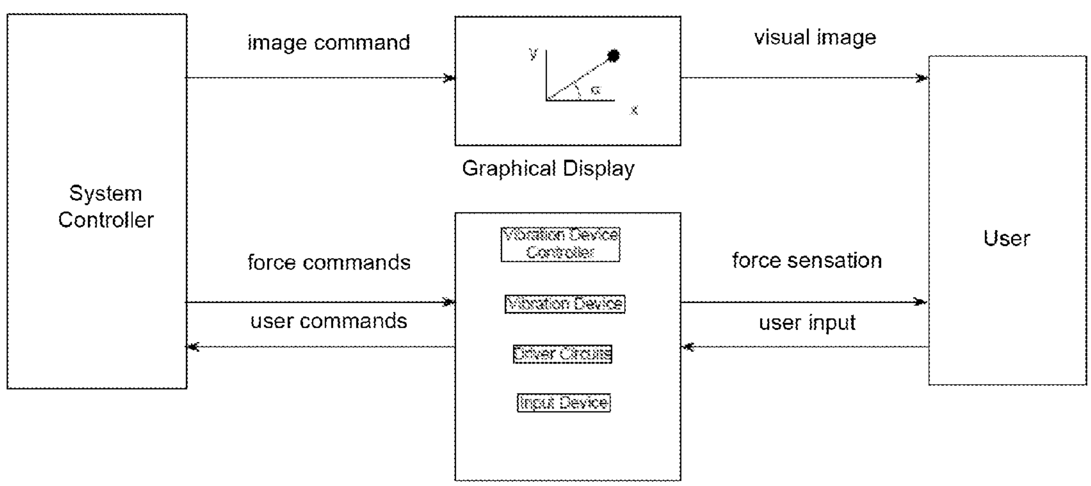

System Controller
image command
y
α
x
Graphical Display
visual image
User
force commands
user commands
Vibration Device Controller
Vibration Device
Driver Circuits
Input Device
force sensation
user input

FIG. 89

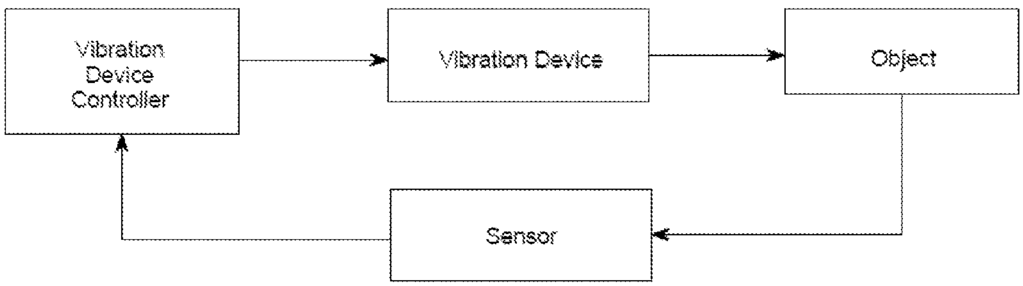
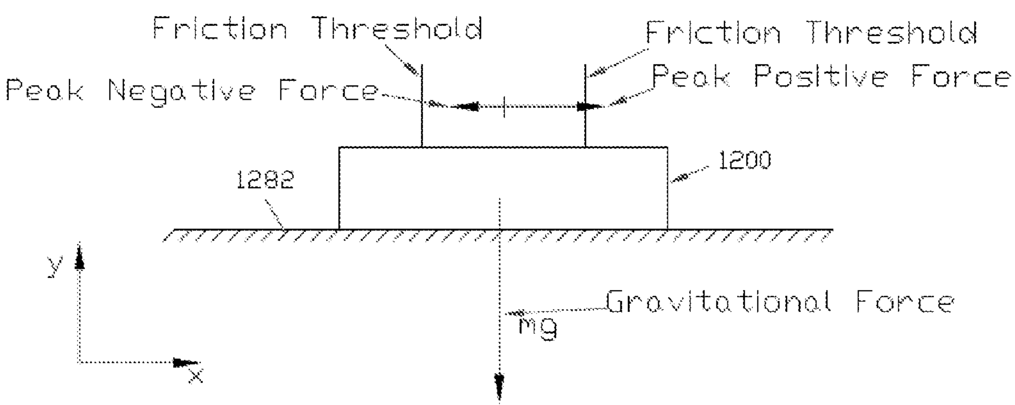

Vibration Device Controller
Vibration Device
Object
Sensor

FIG. 90


Friction Threshold
Friction Threshold
Peak Negative Force
Peak Positive Force
1200
1282
y
x
Gravitational Force
mg

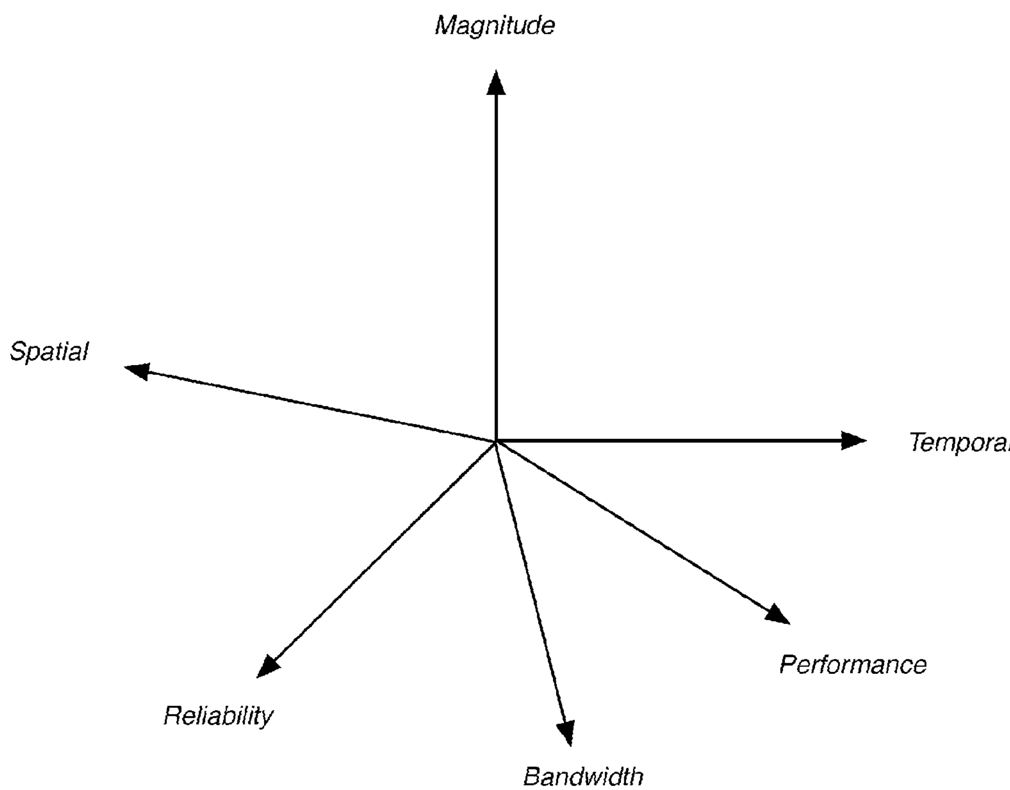

Magnitude
Spatial
Temporal
Performance
Reliability
Bandwidth

FIG. 91

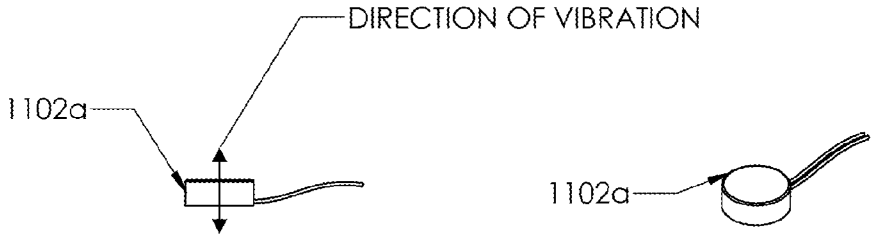


FIG. 92A

FIG. 92B

FIG. 92C

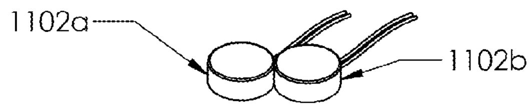


FIG. 92D

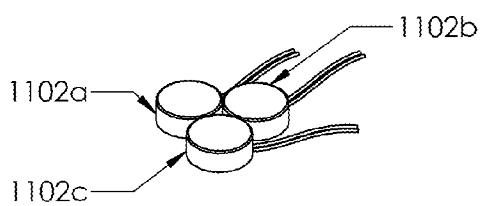


FIG. 92E

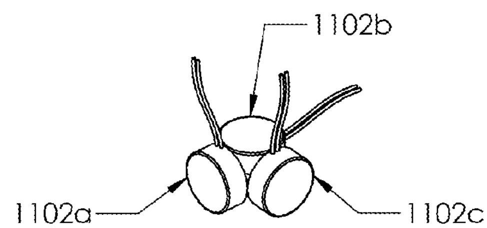


FIG. 92F

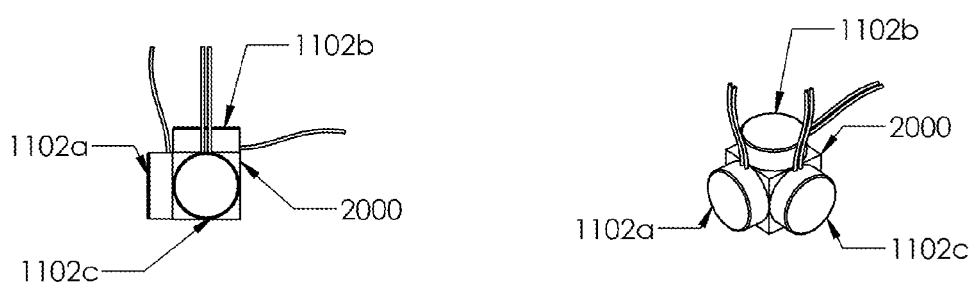


FIG. 92G

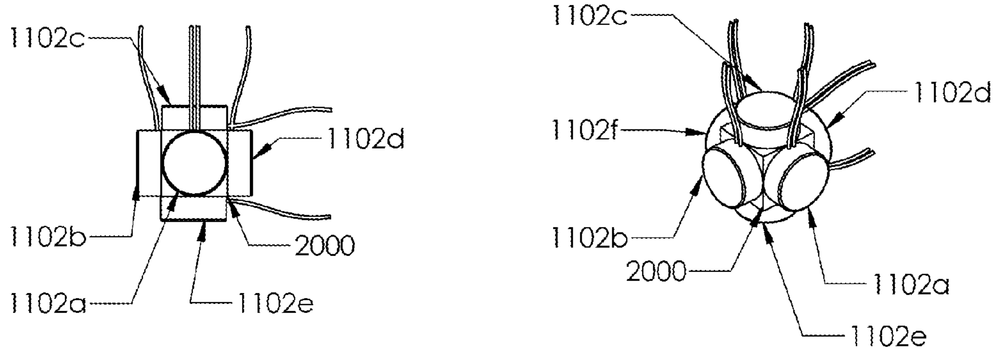


FIG. 92H

FIG. 92I

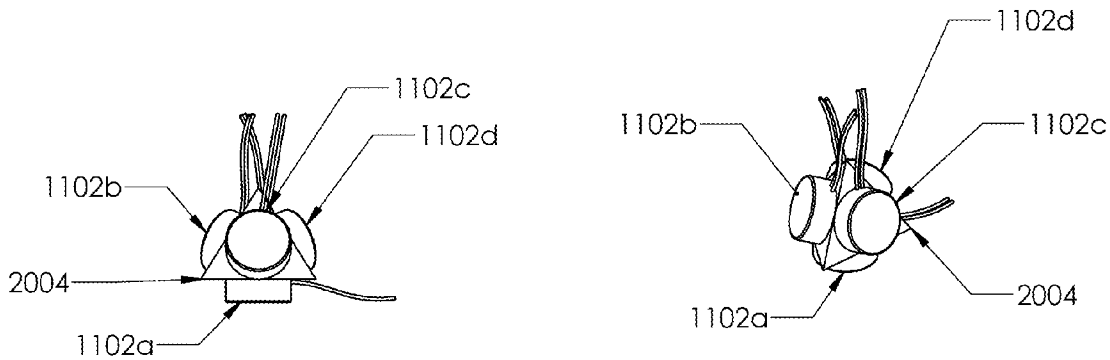


FIG. 92J

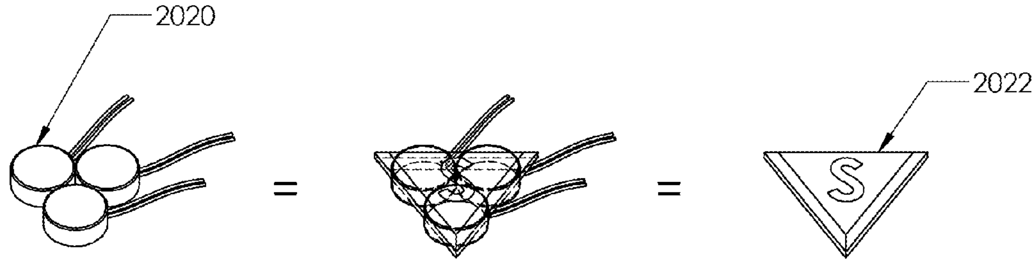


FIG. 93

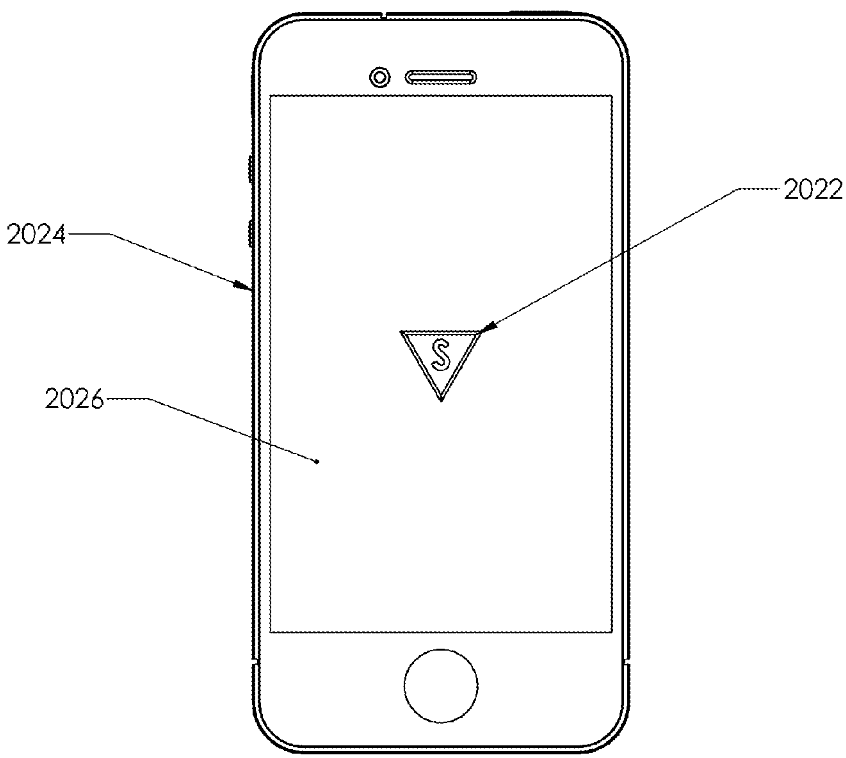


FIG. 94

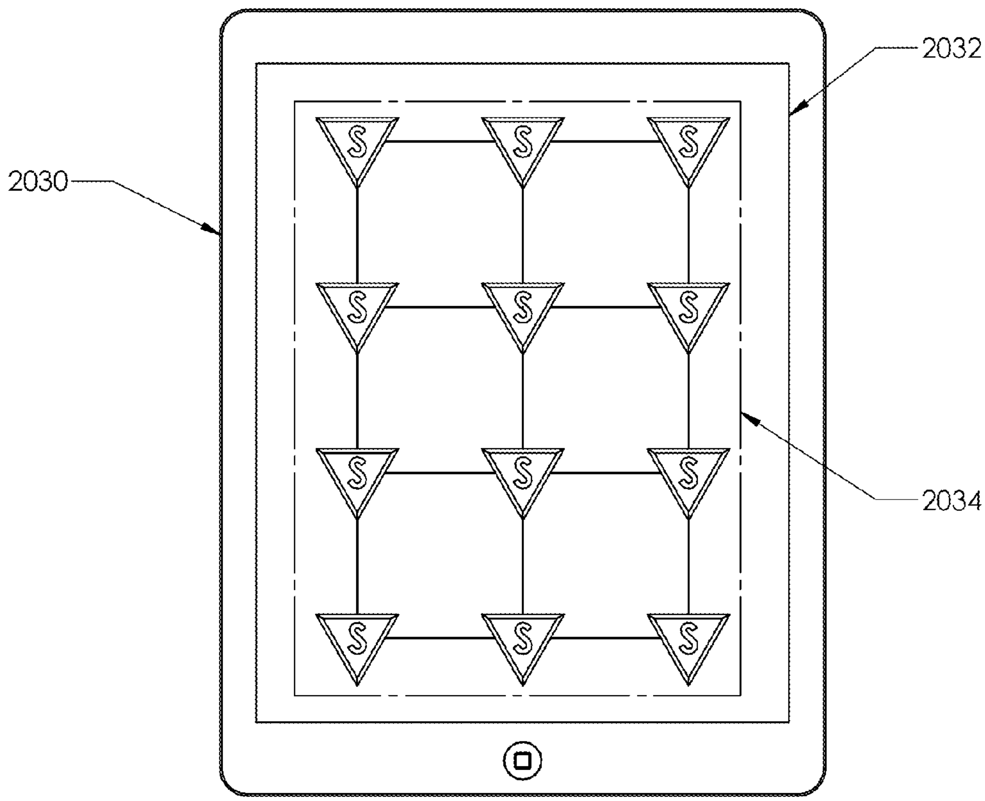
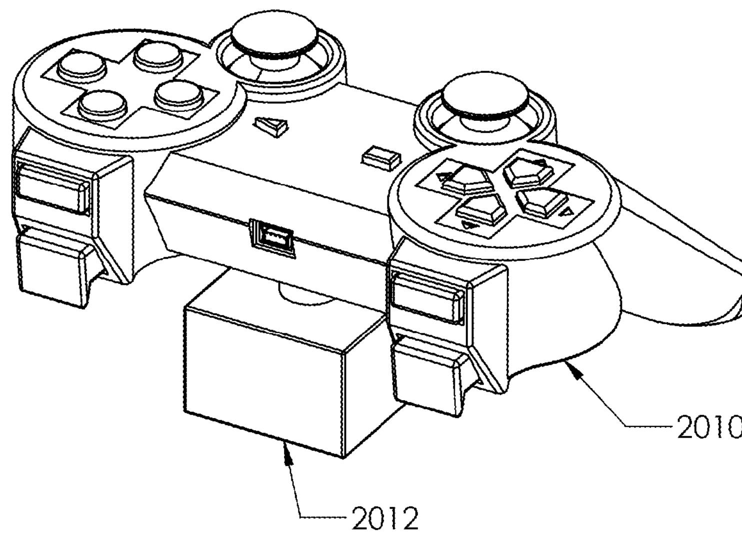


FIG. 95



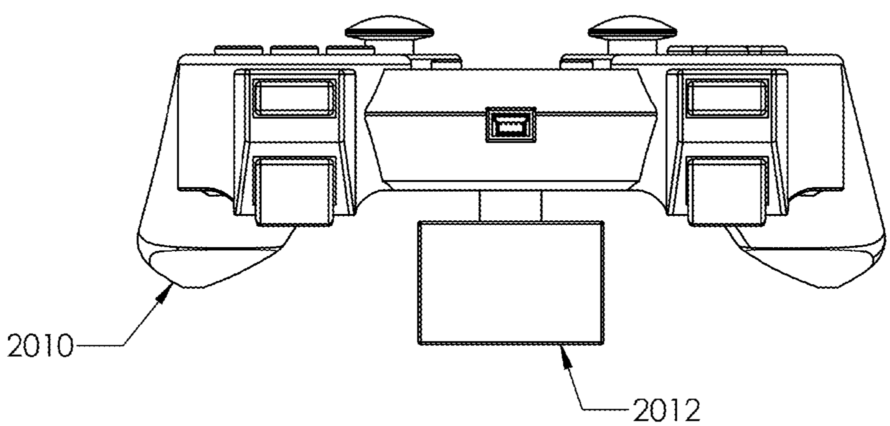


FIG. 96

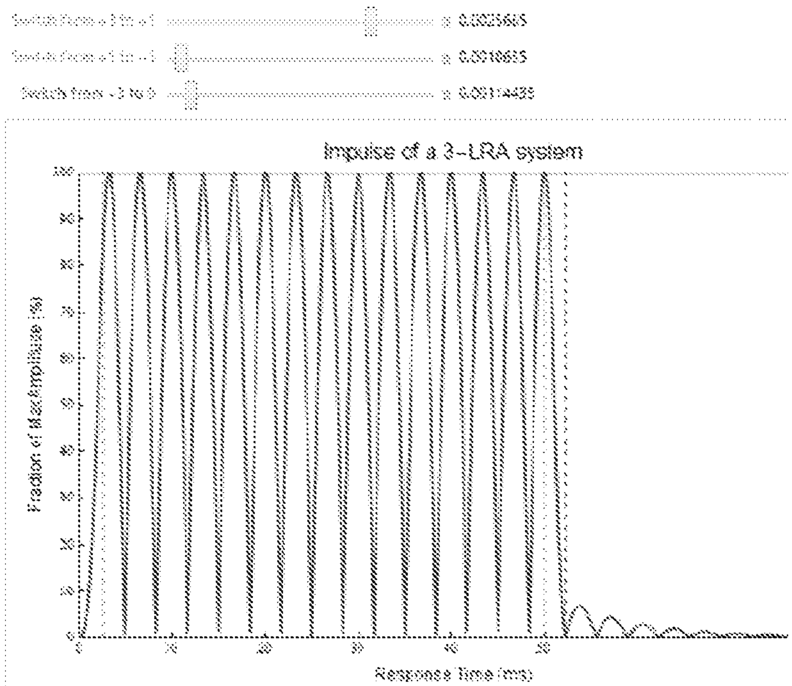

Impulse of a 3-LRA system
Fraction of Max Amplitude (%)
Response Time (ms)

Fig. 97

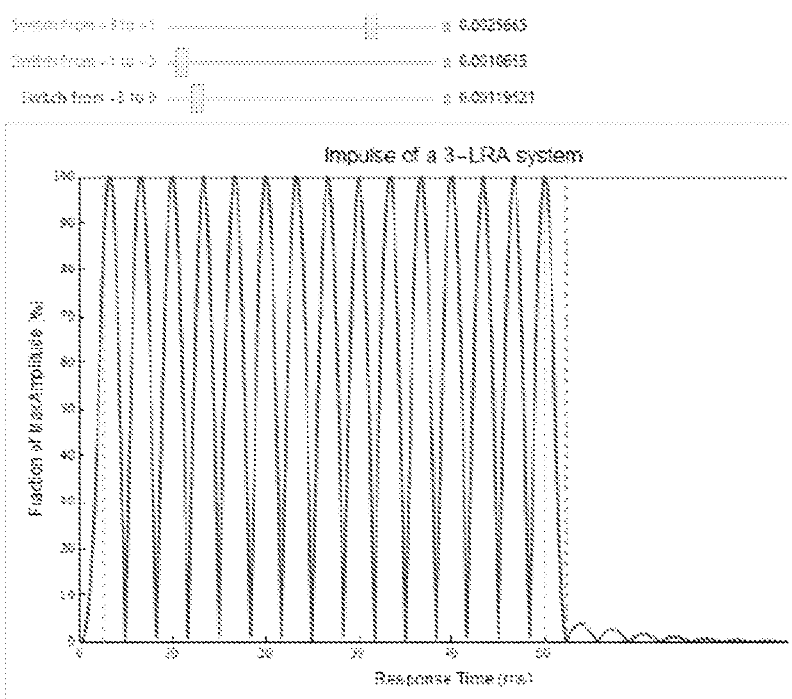

Impulse of a 3-LRA system
Fraction of Max Amplitude (%)
Response Time (ms)

Fig. 98

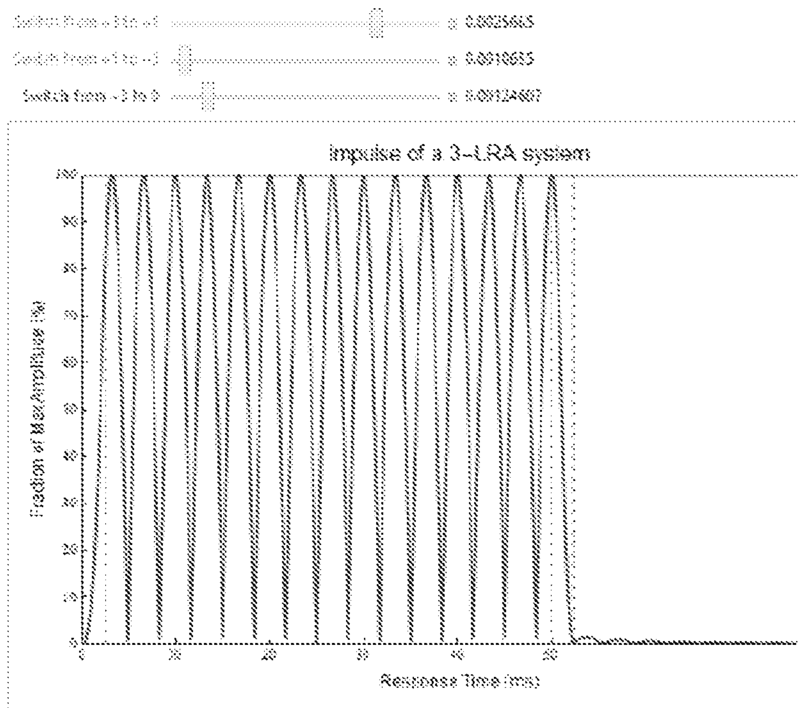


Fig. 99

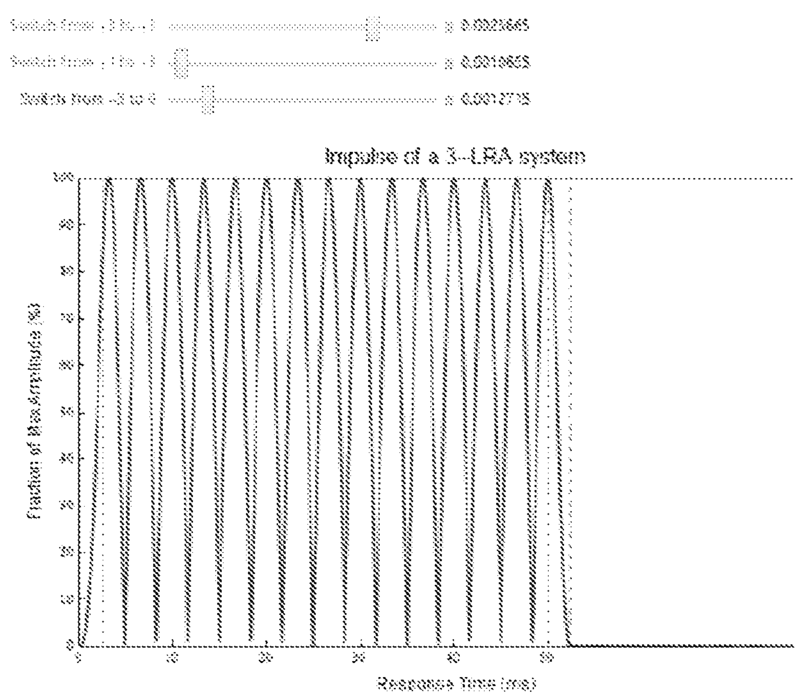


Fig. 100

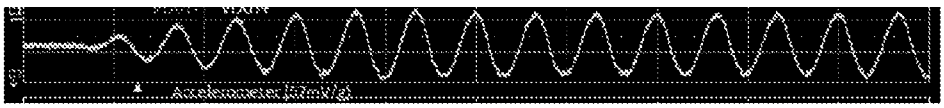


Fig. 101

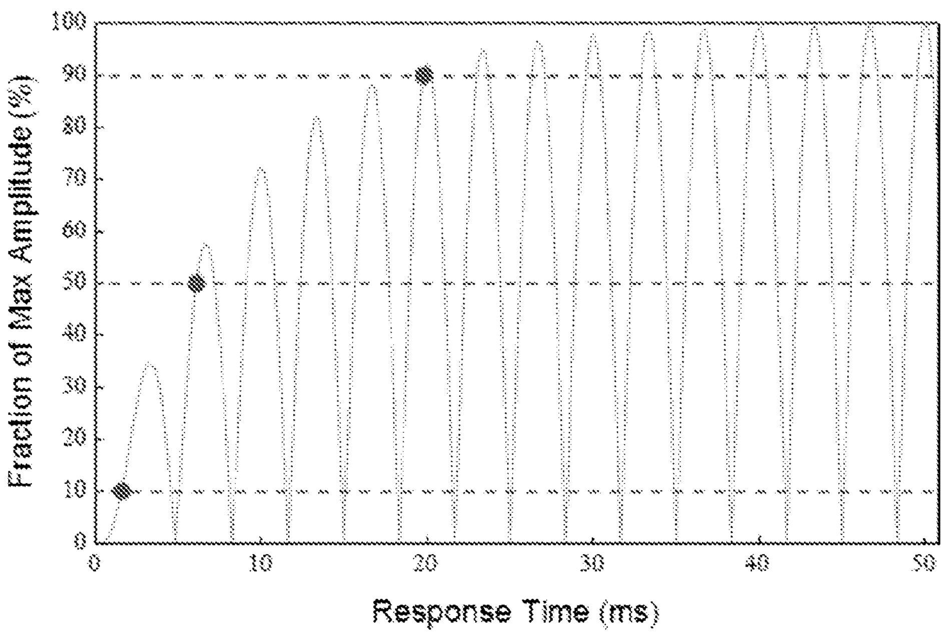


Fig. 102

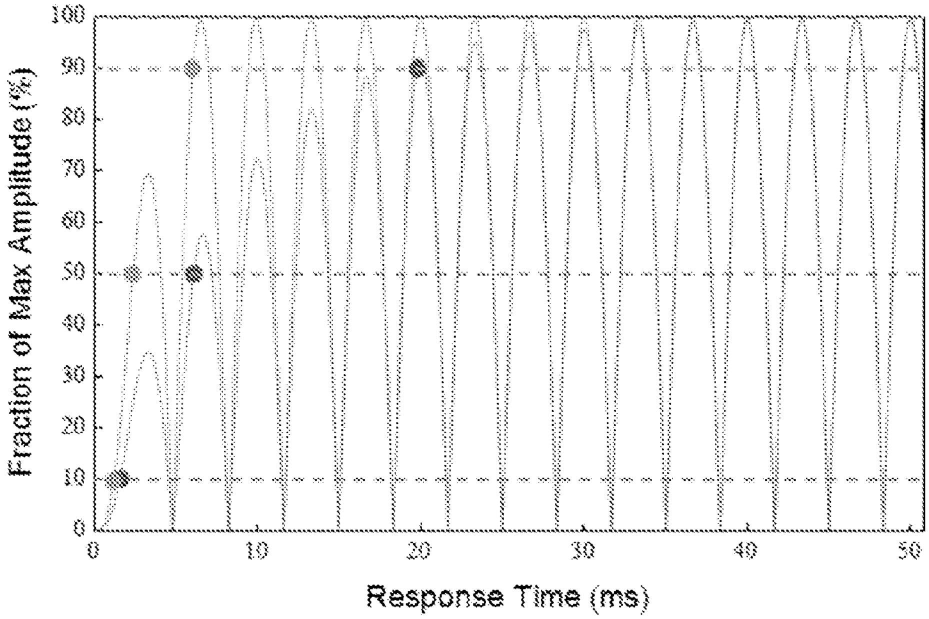


Fig. 103

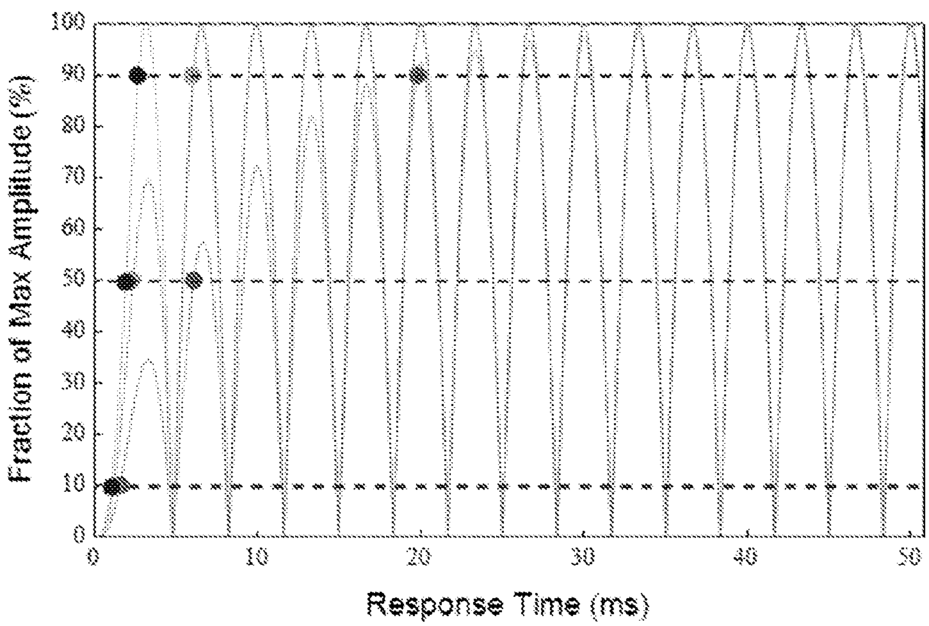


Fig. 104

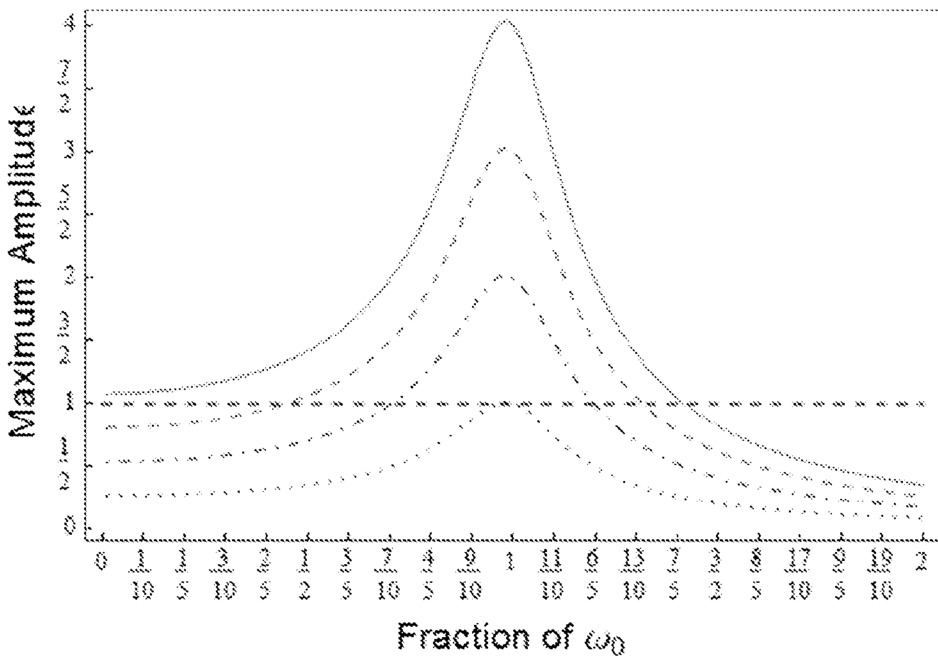

Maximum Amplitude
4
3
2
1
0
0 1/10 1/5 3/10 2/5 1/2 3/5 7/10 4/5 9/10 1 11/10 6/5 13/10 7/5 3/2 8/5 17/10 9/5 19/10 2
Fraction of ω_0

Fig. 105

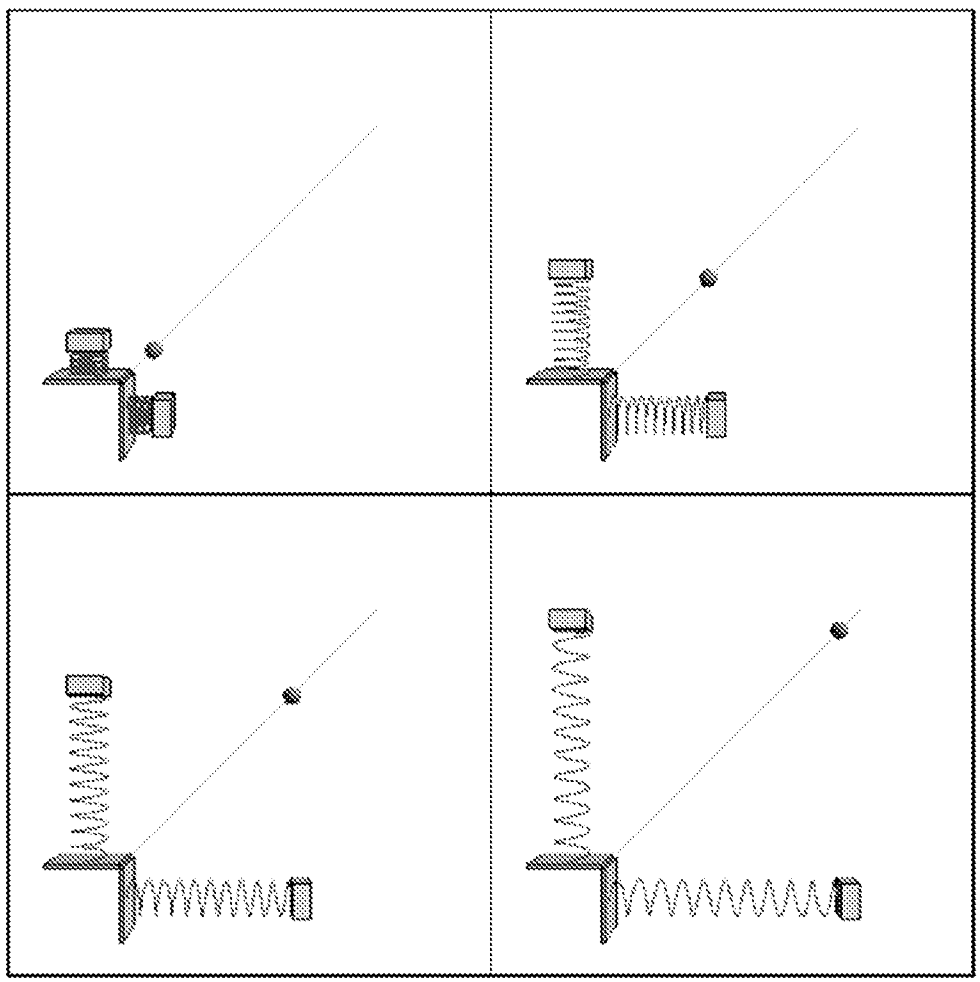


Fig. 106

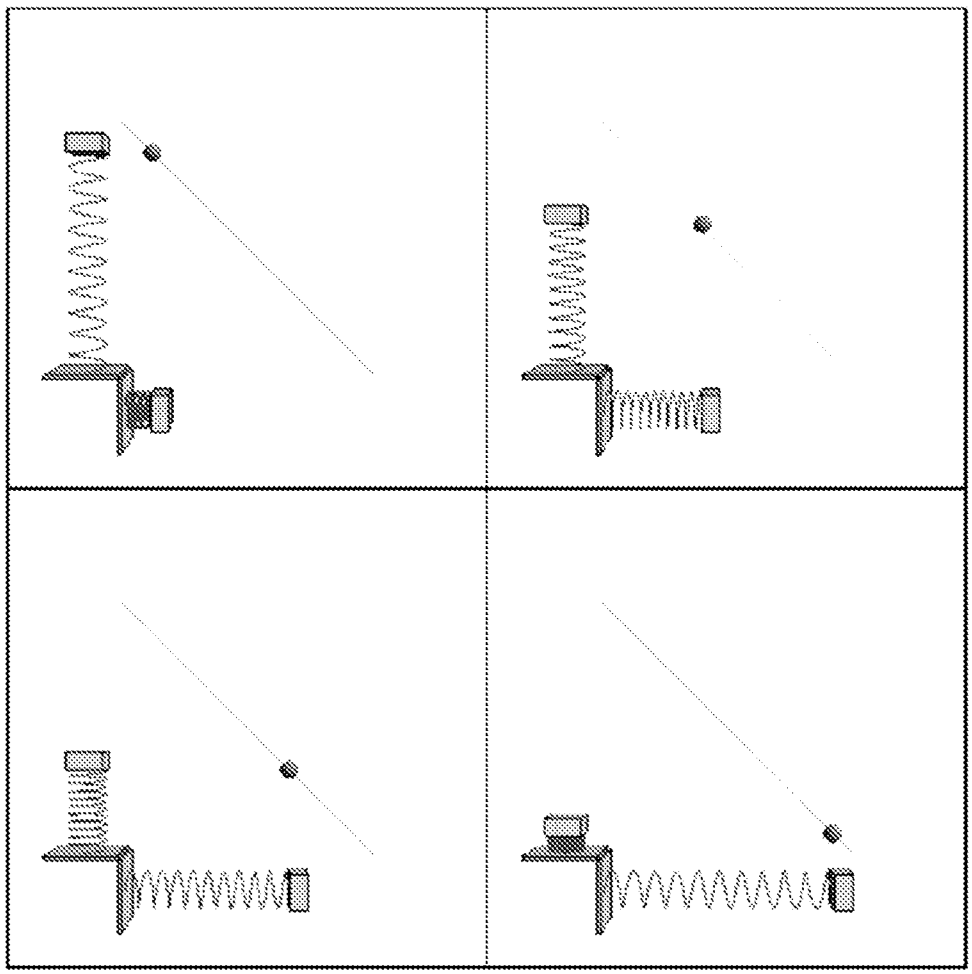


Fig. 107A

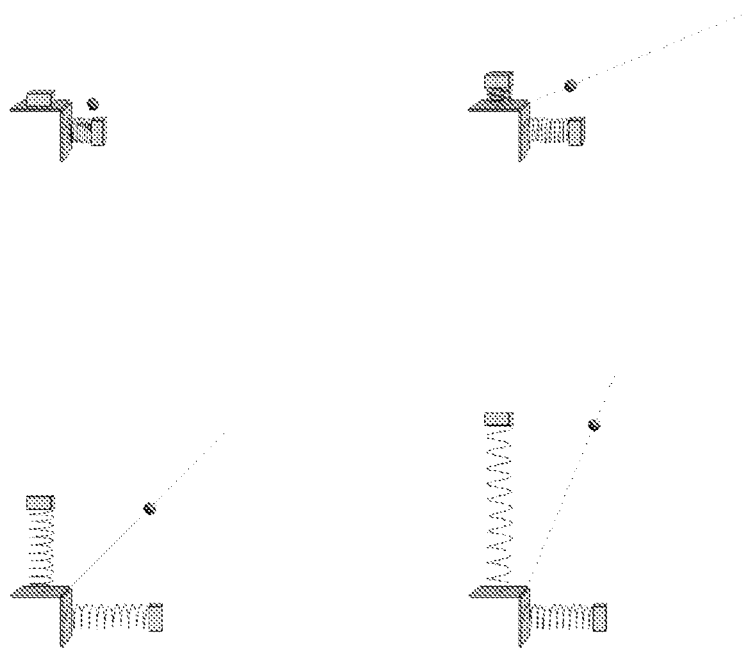


Fig. 107B

Note: this figure is missing a box with four cells to contain the four sub-drawings.

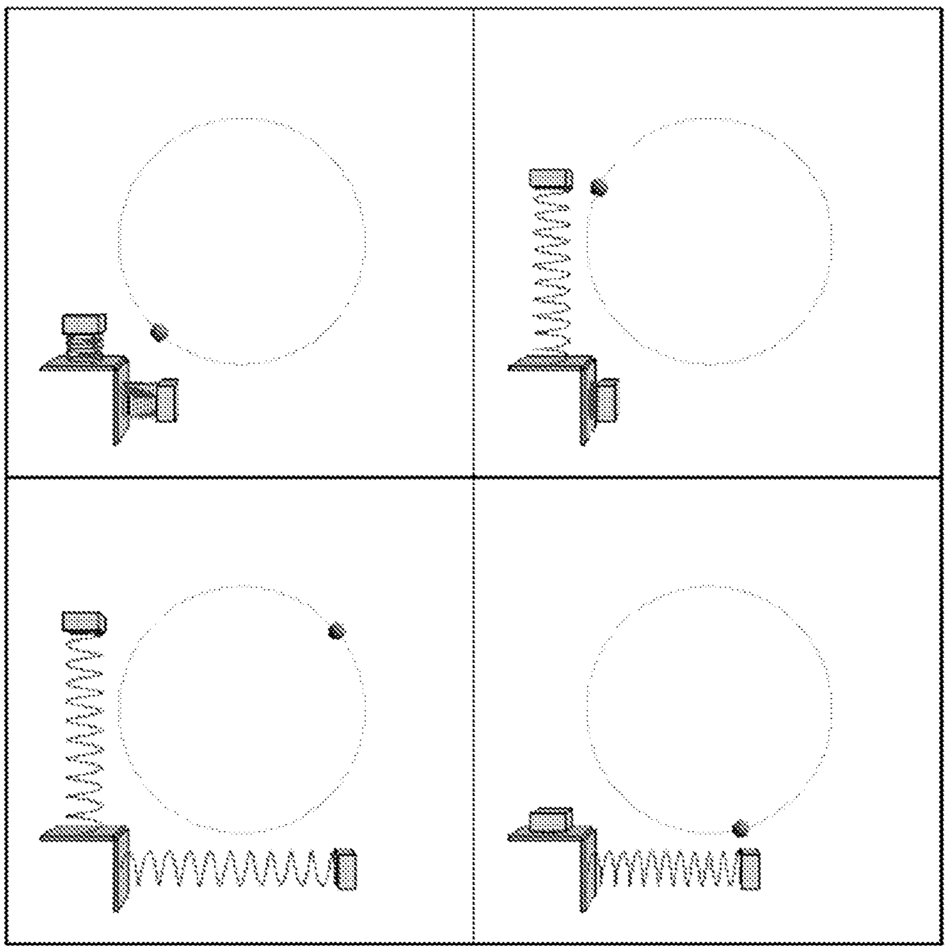


Fig. 108A

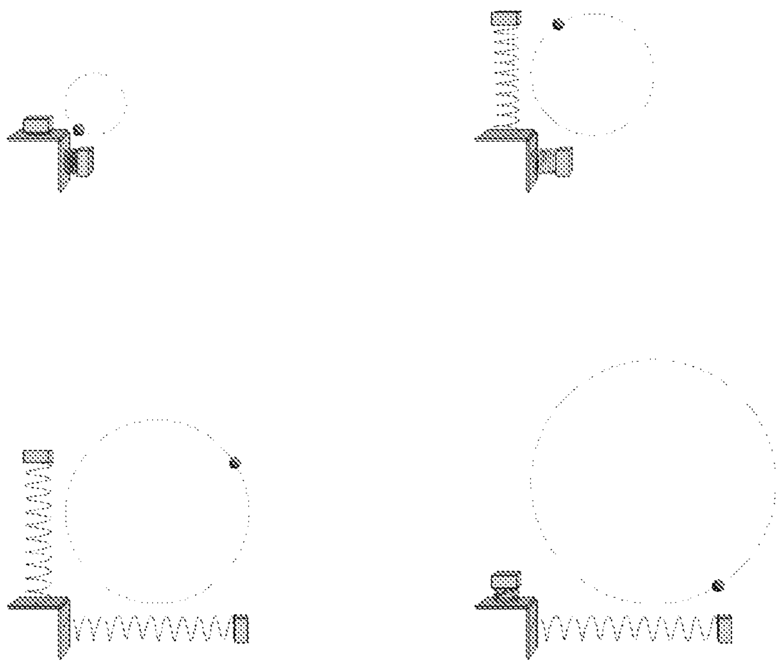


Fig. 108B

Note: this figure is missing a box with four cells to contain the four sub-drawings.

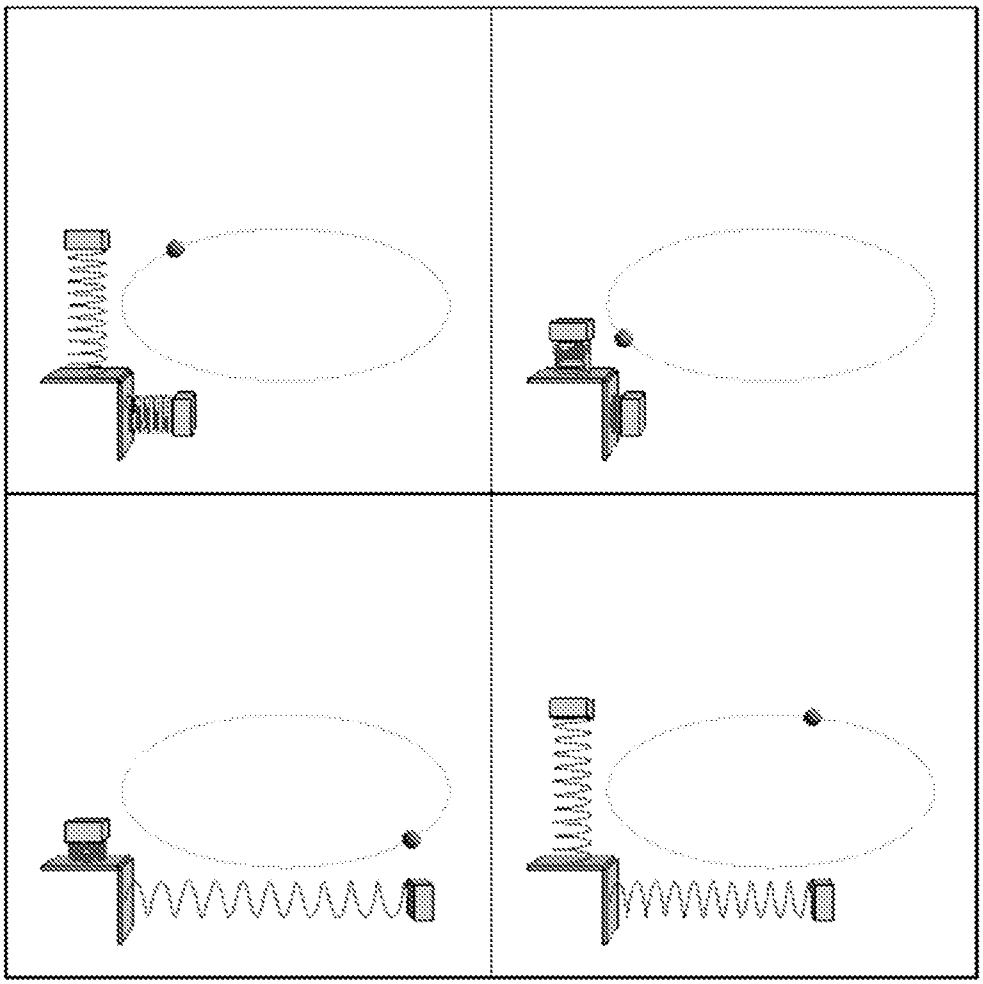


Fig. 109

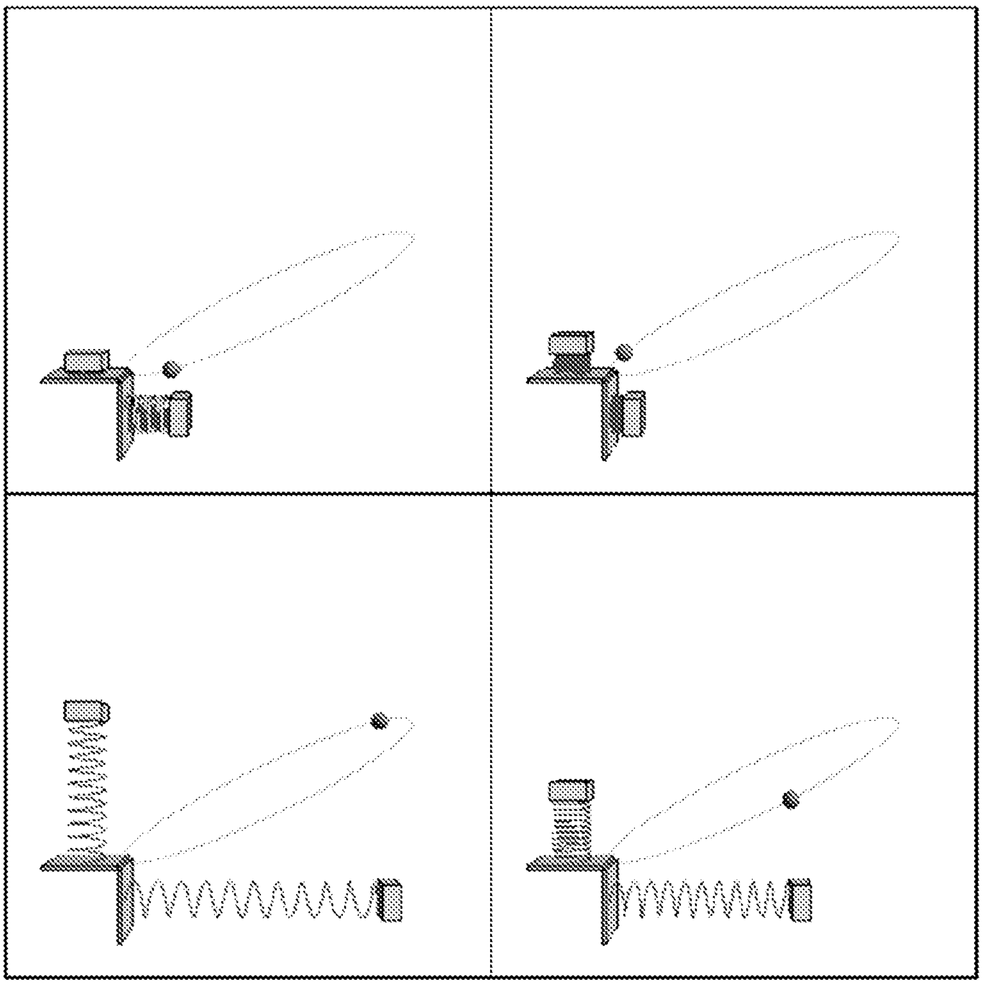


Fig. 110A

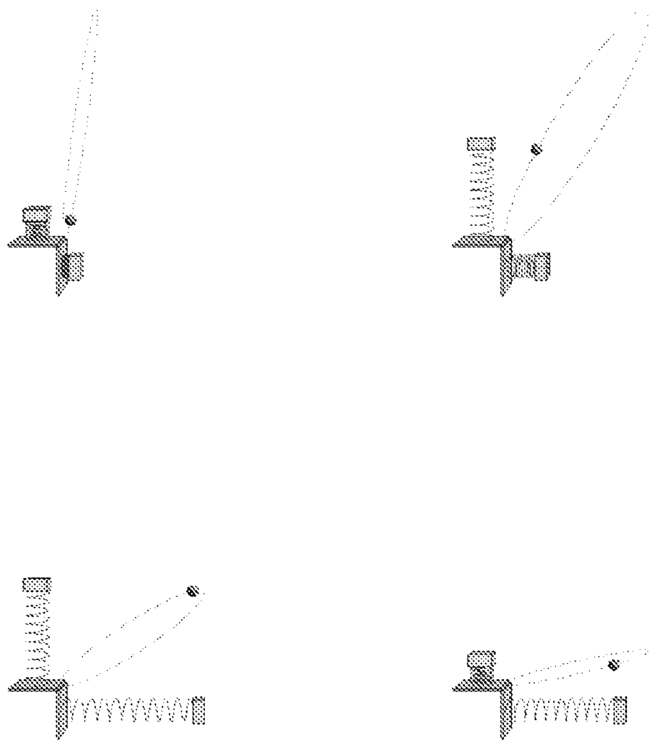


Fig. 110B

Note: this figure is missing a box with four cells to contain the four sub-drawings.

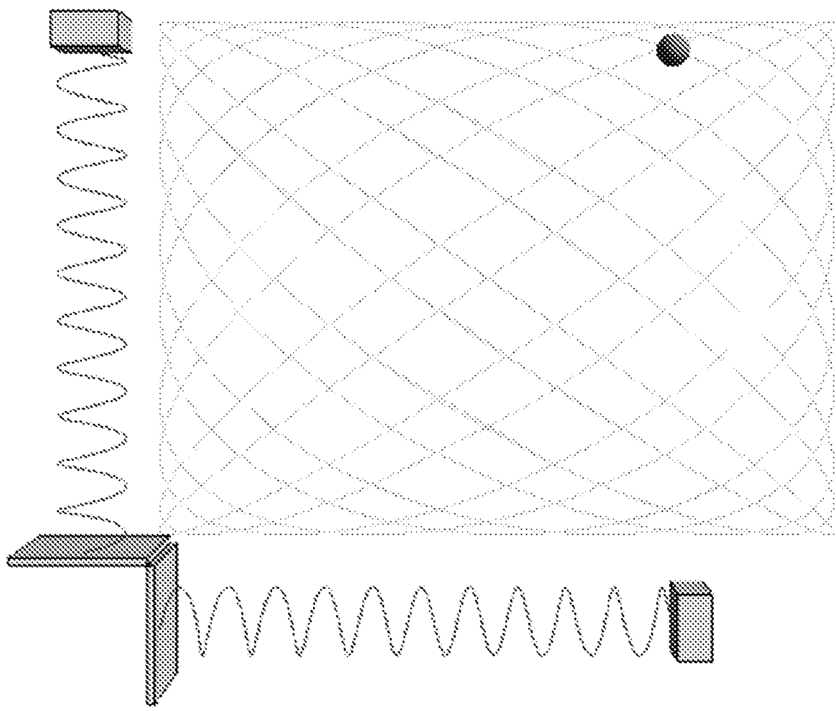


Fig. 111

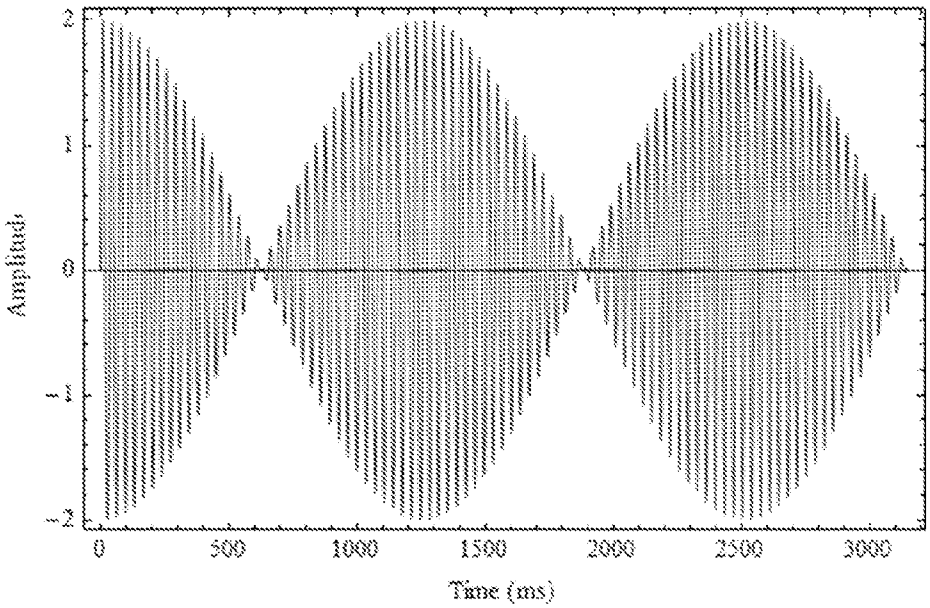


Fig. 112

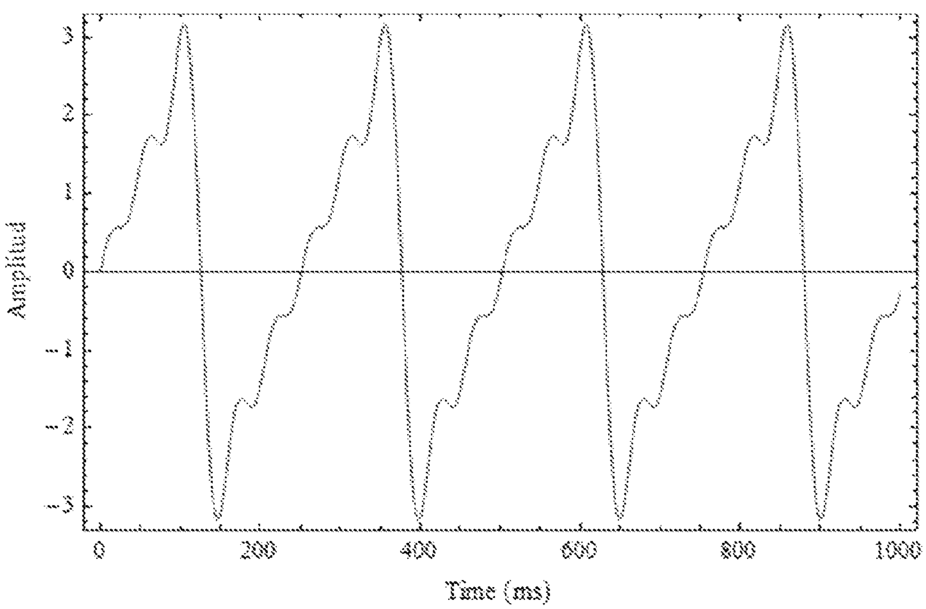


Fig. 113

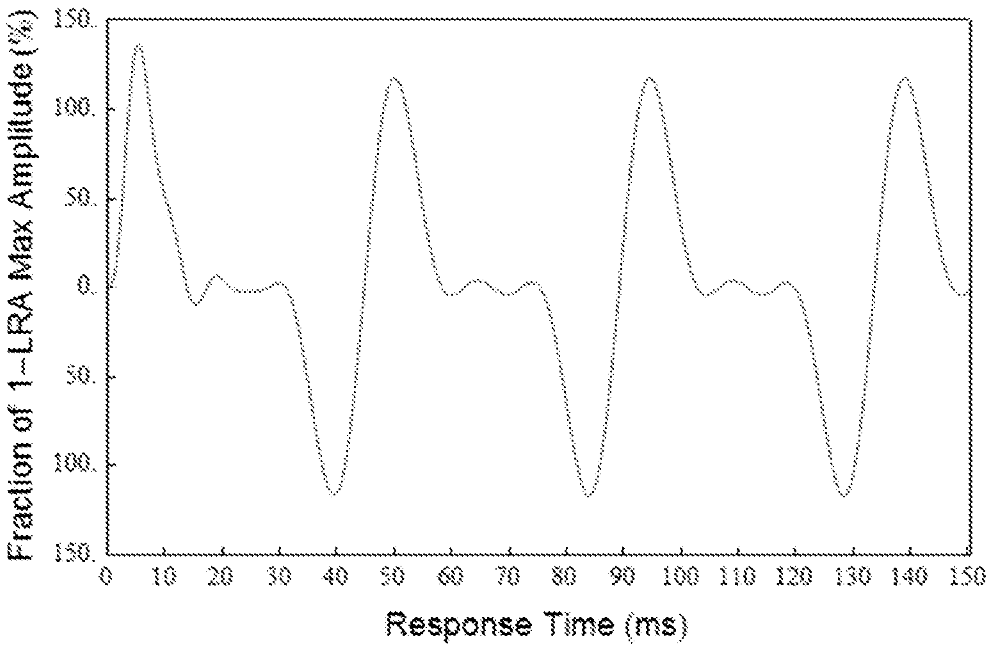


Fig. 114

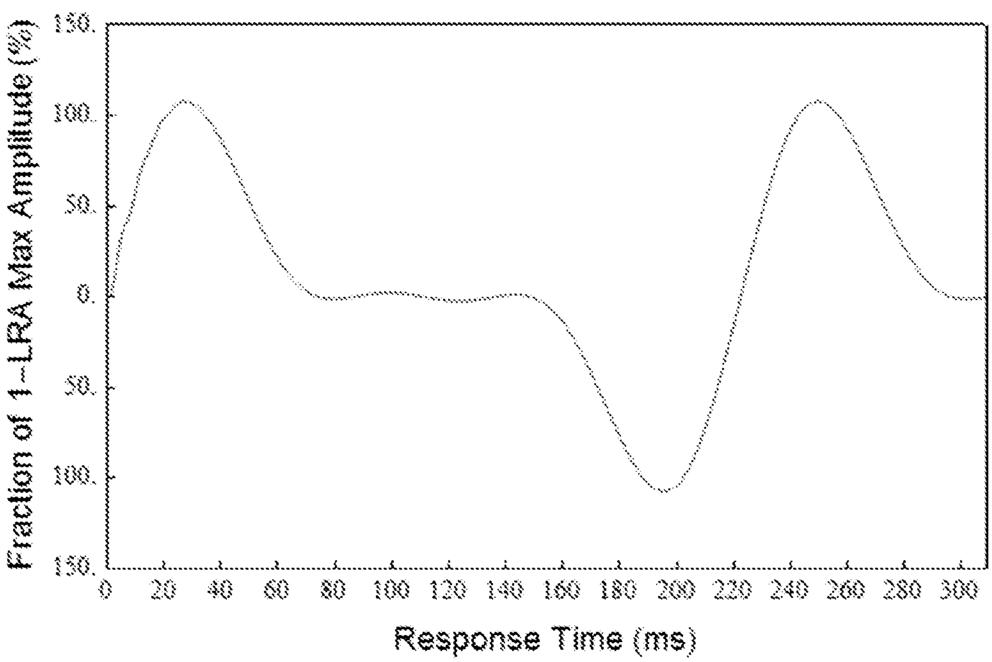


Fig. 115

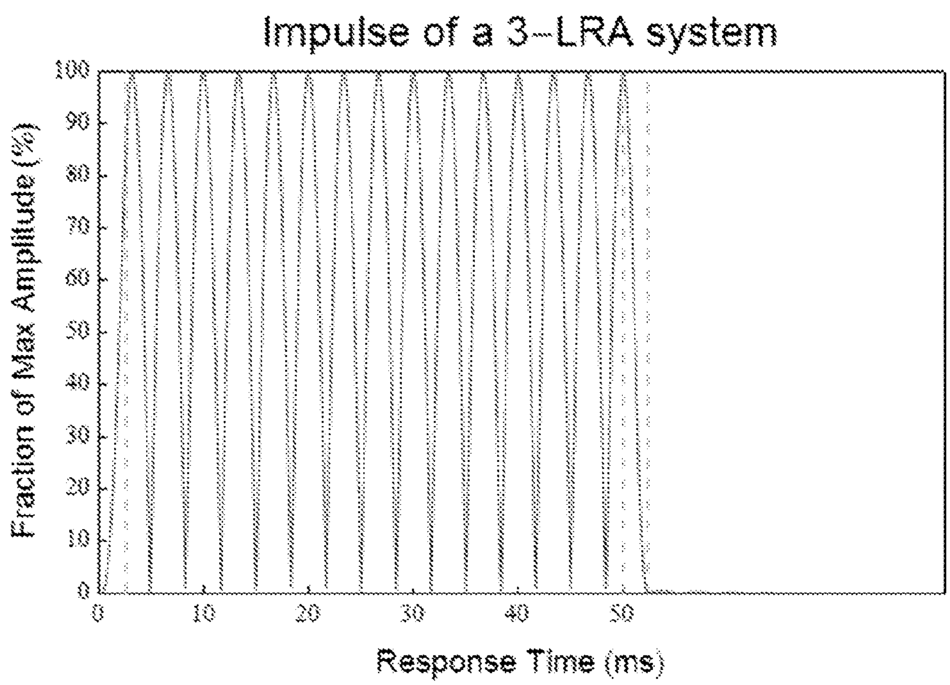


Fig. 116

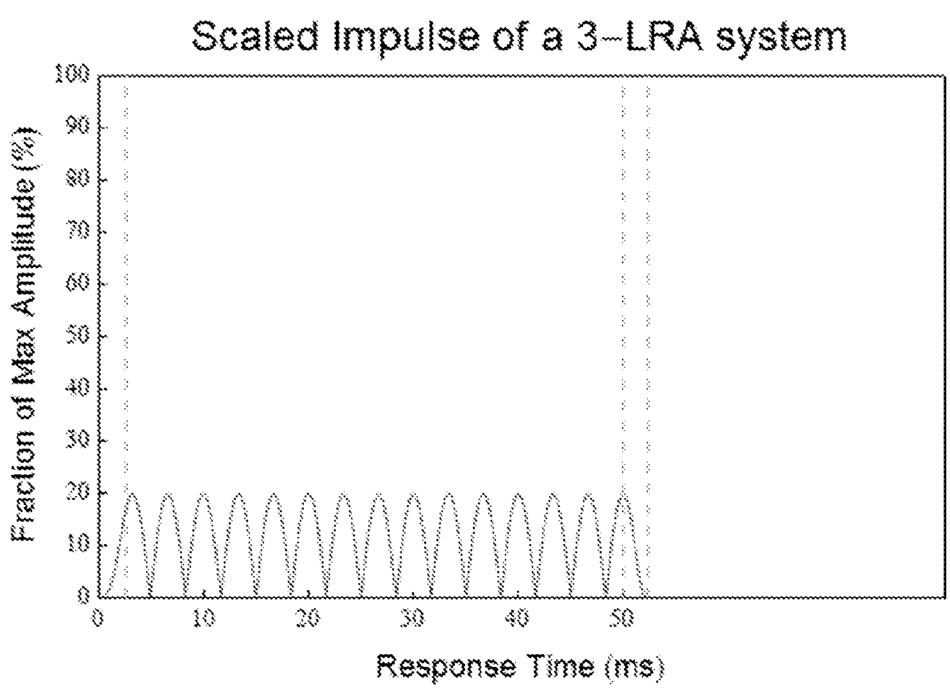


Fig. 117

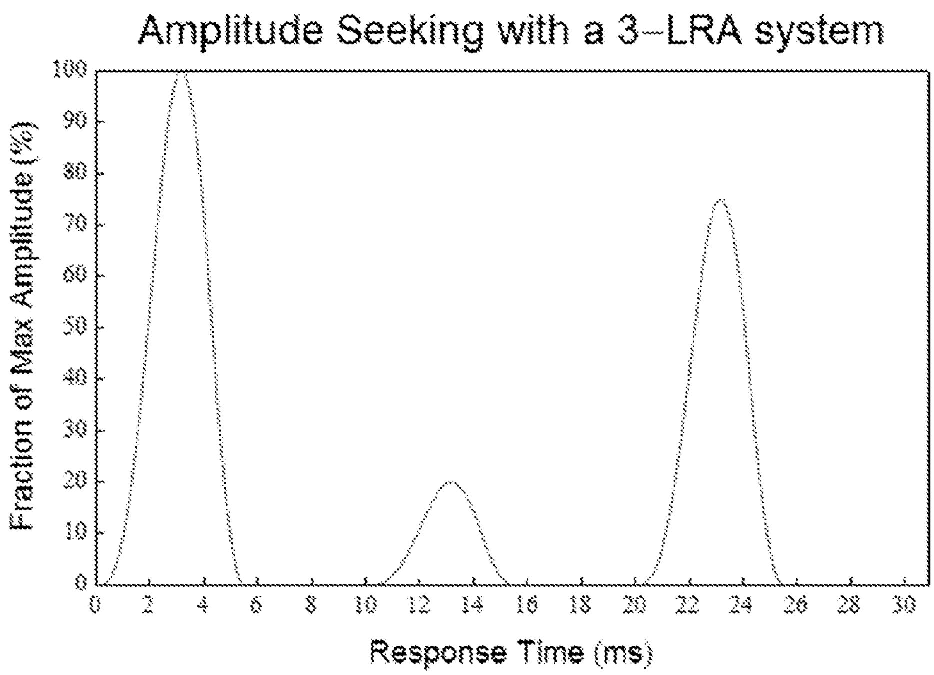


Fig. 118

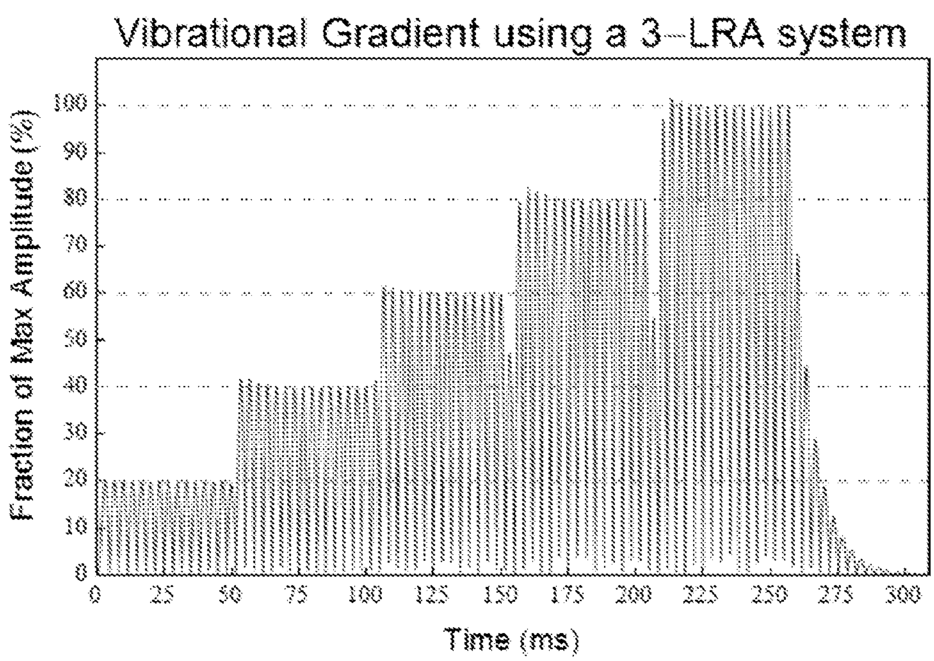


Fig. 119

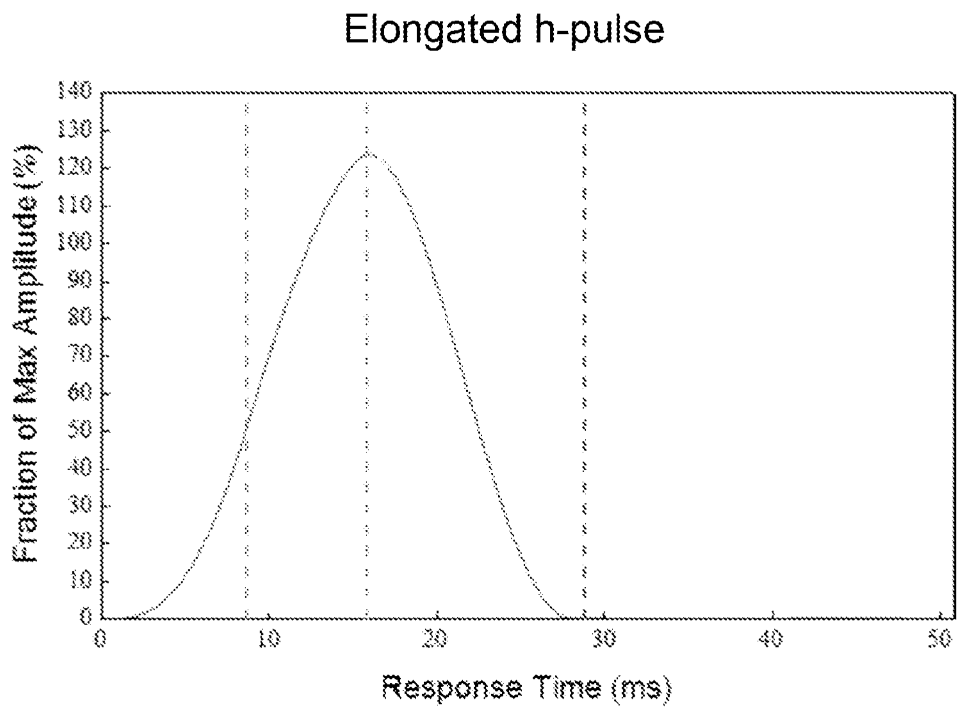


Fig. 120

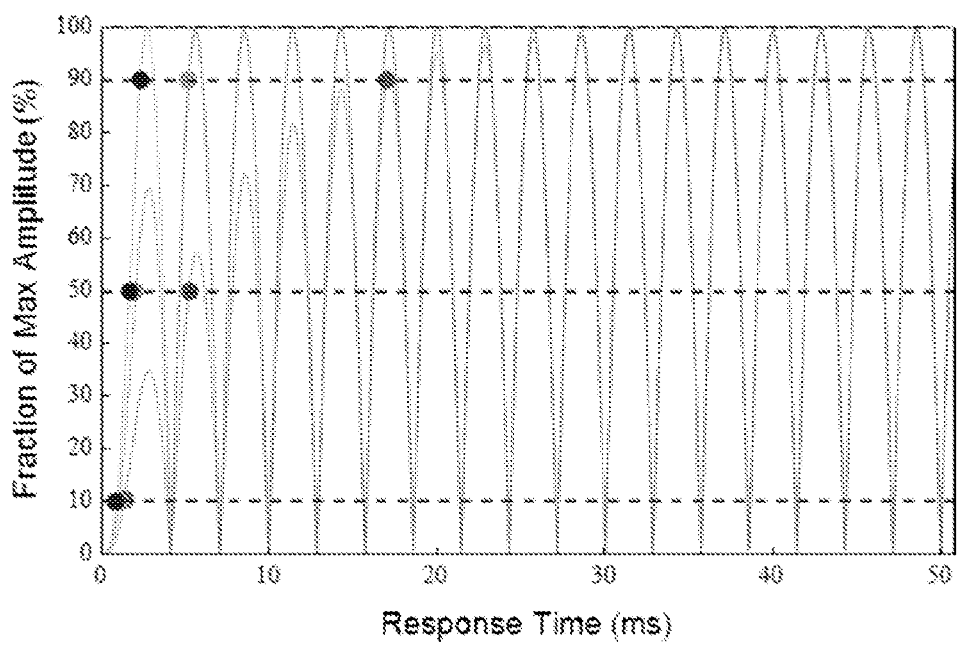


Fig. 121

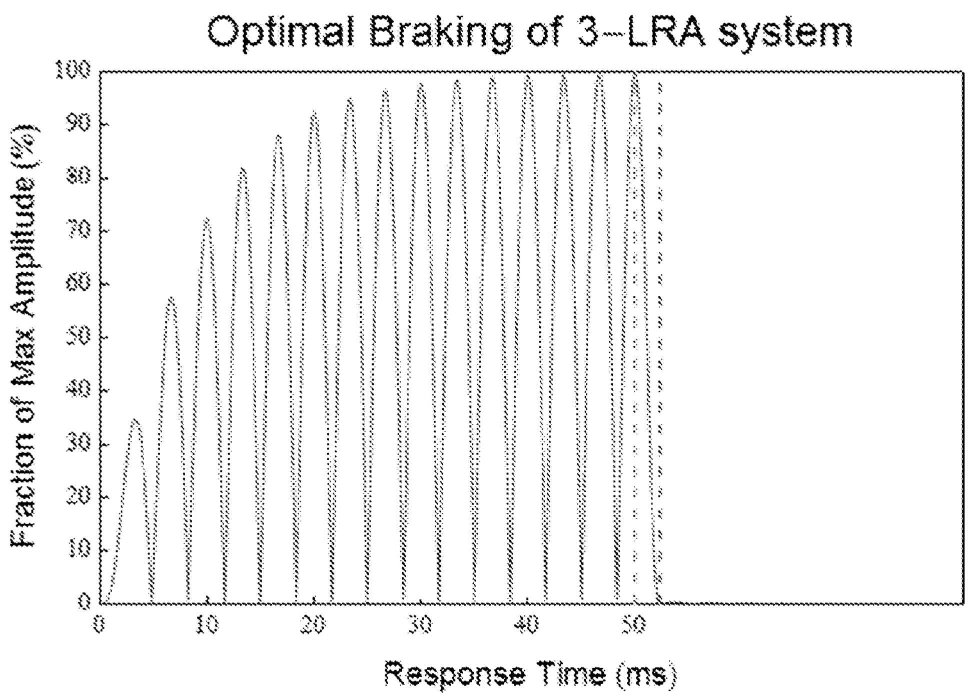

Optimal Braking of 3-LRA system
Fraction of Max Amplitude (%)
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
Response Time (ms)

Fig. 122

SYNCHRONIZED ARRAY OF VIBRATION ACTUATORS IN A NETWORK TOPOLOGY

CROSS REFERENCE TO RELATED APPLICATIONS

The present application is a national phase entry under 35 U.S.C. §371 of International Application No. PCT/US2013/029375 filed Mar. 6, 2013, published in English, which claims the benefit of the filing date of U.S. Provisional Patent Application No. 61/607,092, filed Mar. 6, 2012 and entitled SYNCHRONIZED ARRAY OF VIBRATION ACTUATORS IN A NETWORK TOPOLOGY, the entire disclosures of which are hereby expressly incorporated by reference herein. This application is related to U.S. patent application Ser. No. 13/422,453, filed Mar. 16, 2012 and entitled ASYMMETRIC AND GENERAL VIBRATION WAVEFORMS FROM MULTIPLE SYNCHRONIZED VIBRATION ACTUATORS, which is a continuation-in-part of U.S. patent application Ser. No. 13/030,663, filed Feb. 18, 2011, and entitled SYNCHRONIZED VIBRATION DEVICE FOR HAPTIC FEEDBACK, which is a continuation of U.S. application Ser. No. 11/476,436, filed Jun. 27, 2006, issued on Apr. 5, 2011 as U.S. Pat. No. 7,919,945, which claims the benefit of the filing date of U.S. Provisional Patent Application No. 60/694,468 filed Jun. 27, 2005 and entitled SYNCHRONIZED VIBRATION DEVICE FOR HAPTIC FEEDBACK, the entire disclosures of which are hereby expressly incorporated by reference herein. This application is also related to U.S. Provisional Patent Application No. 61/453,739, filed Mar. 17, 2011 and entitled ASYMMETRIC AND GENERAL VIBRATION WAVEFORMS FROM MULTIPLE SYNCHRONIZED VIBRATION ACTUATORS, and to U.S. Provisional Patent Application No. 61/511,268, filed Jul. 25, 2011 and entitled ASYMMETRIC AND GENERAL VIBRATION WAVEFORMS FROM MULTIPLE SYNCHRONIZED VIBRATION ACTUATORS, the entire disclosures of which are hereby expressly incorporated by reference herein.

BACKGROUND OF THE INVENTION

Vibration devices are used in a wide range of applications including haptic displays, haptic interfaces, force feedback devices, vibratory feeders, beauty products, personal hygiene products, personal pleasure products, personal massagers, tree harvesters, and seismic vibrators. Some widely used products that include haptic displays include the DUALSHOCK® 3 wireless controller for Sony Computer Entertainment's PlayStation® 3; the PlayStation® Move motion controller for motion gaming with Sony Computer Entertainment's PlayStation® 3; Microsoft Corporation's Xbox 360 Wireless Speed Wheel; and the Wii Remote™ Plus controller which is used for motion gaming with the Nintendo Wii.

Vibration actuators are typically the smallest and lowest cost method for generating haptic sensations. Therefore, it is advantageous to use vibration actuators to create a wide range of haptic sensations. Common low cost vibration actuators include Eccentric Rotating Mass actuators (ERMs) and Linear Resonant Actuators (LRAs). One of the advantages of both ERMs and LRAs is that they can generate relatively large vibration forces from low power input. Both ERMs and LRAs generally build up kinetic energy during their ramp-up period; an ERM does this as the velocity of its rotating mass increases, and an LRA does this as the amplitude of vibration of its moving mass increases. These low cost actuators are used in many applications, including in consumer electronics products such as smartphones and videogame controllers.

Many smartphones today use either a single ERM or a single LRA to produce alerts by vibrating the entire device. This has the advantage that the vibration alert can be felt while the device is inside a person's pocket. Game controllers (also commonly termed interchangeably as "videogame controllers" or simply "controllers") often incorporate two ERMs within a two-handed device such as the Xbox 360 Wireless Controller or the Xbox 360 Wireless Speed Wheel (both devices from Microsoft). Sometimes such dual-ERM controllers are configured with one ERM having a large rotating mass and the other ERM having a small rotating mass. A single-handed controller such as the Wii Remote™ Plus (from Nintendo) will typically have a single ERM to provide vibration feedback to the user.

A common limitation of most existing vibration devices is the inability to define the directionality of the vibratory forces. ERM actuators generate centripetal forces that rotate in a plane, and generally the direction of vibration (that is to say, the instantaneous direction of the rotating centripetal force vector) cannot be not sensed in haptic applications due in part to the high rate of change of the direction of vibrations. In an ERM a centripetal force is applied onto the eccentric mass by the motor shaft, and an equal and opposite centrifugal force is applied onto the motor shaft. In this document both the terms centripetal and centrifugal are used with the understanding that these are equal but opposite forces. LRAs vibrate back and forth, and thus it may be possible to sense the axis of vibration, but it is not possible to provide more of a sensation in the forward direction relative to the backward direction or vice versa. Since haptic applications are often integrated with audio and video displays such as in computer gaming where directions are an integral component of the game, it is desirable to provide a haptic sensation that also corresponds to a direction. Moreover, it is be useful to generate haptic cues of directionality for applications where a person does not have visual cues, such as to guide a vision-impaired person. Therefore, it is desirable to provide a human-perceptible indication of directionality in vibratory haptic displays and interfaces. In addition, it is advantageous to use vibration actuators to generate a wide range of vibration waveforms including both directional and non-directional waveforms.

There have been some haptic vibration devices that provide a sensation of vibration direction, but these prior implementations have disadvantages. Specifically, asymmetric vibrations have been used to generate a haptic sensation that is larger in one direction than the opposite direction.

However, existing asymmetric vibrators are complex, costly, or have limited controllability. They tend to be bulky and have low power efficiency. Tappeiner et. al. demonstrated a vibration device that generated asymmetric directional haptic cues (Tappeiner, H. W.; Klatzky, R. L.; Unger, B.; Hollis, R., "Good vibrations: Asymmetric vibrations for directional haptic cues", World Haptics 2009, Third Joint Euro Haptics Conference and Symposium on Haptic Interfaces for Virtual Environments and Teleoperator Systems), yet this device uses a high power and an expensive 6-DOF magnetic levitation haptic device. Amemiya et. al. (Tomohiro Amemiya; Hideyuki Ando; Taro Maeda; "Kinesthetic Illusion of Being Pulled Sensation Enables Haptic Navigation for Broad Social Applications, Ch. 21, Advances in Haptics, pp. 403-414") illustrated a device that also generates asymmetric vibrations for haptic applications, yet this device uses a complex and large linkage system with 6 links and it appears that the direction of vibration cannot be modified in real-time.

Another limitation of vibration devices that use ERMs is that the amplitude of vibration is dependent on the frequency of vibration, since the vibration forces are generated from centripetal acceleration of an eccentric mass. Some prior approaches have used multiple ERMs to control frequency and amplitude independently, but in the process also generate undesirable torque effects due to the offset between the ERMs.

SUMMARY OF THE INVENTION

It is desirable to produce not only haptic directional cues but also to be able to render legacy haptic effects, so that a single game controller could generate both new and existing sensations. Therefore, one aspect of this disclosure and associated embodiments described in this specification include vibration devices and methods of operating those devices employing multiple, low-cost actuator components in lieu of a single, high-cost actuator for creating haptic effects. The aspects described within this disclosure include synchronizing arrays of low-cost, readily available vibration actuators to emulate superlative single actuators and to bring together sets of these emulated high-performance actuators to create a broad range of desired control effects.

According to one aspect of the invention, a vibration device comprises a mounting platform, a plurality of linear resonant actuators attached to the mounting platform, and a controller. Each of the plurality of linear resonant actuators has a moveable mass and an axis of vibration in accordance with a direction of movement of the moveable mass. The axes of vibration of the plurality of actuators being arranged in a substantially parallel configuration. The controller is coupled to each of the plurality of actuators, and is configured to (1) control each actuator to impart a sinusoidal vibration force of a frequency, f_1, onto the mounting platform, and (2) control amplitudes and phases of the sinusoidal vibration force from each actuator to generate a combined vibration waveform onto the mounting platform. The combined vibration waveform emulates a virtual vibration actuator having one or both of the following properties: a faster amplitude response than any one of the plurality of linear resonant actuators, or a larger amplitude response than any one of the plurality of linear resonant actuators for non-resonant frequencies.

In one example, all of the plurality of linear resonant actuators are substantially identical. In another example, the vibration device is configured for haptic applications. In one alternative, the vibration device is further configured to be portable by a person. Here, the vibration device may be further configured to be worn or held by a person.

In any of the above examples, the vibration device may be further configured to provide haptic output for one of the following devices: a game controller, a motion game controller, a handheld game console, a remote control, a handheld portable computer, a navigation device, a handheld construction tool, a handheld surgical tool, a stylus, a plush toy, a pair of eyeglasses, a wristband, a wristwatch, a belt, an armband, a leg band, a mobile phone, a tablet computer, a device for aiding a vision-impaired person, a device for aiding a hearing-impaired person, and a device for augmenting reality with haptic feedback, a personal pleasure device for providing pleasurable haptic sensations, and a vibration device used for singly or in a pair for conveying telepresence.

In another example, the combined waveform has the larger amplitude response than any one of the linear resonant actuators for non-resonant frequencies, and the virtual vibration actuator has a control sequence that controls the combined waveform to go from zero amplitude to maximum amplitude within one quarter-wavelength. In a further example, the virtual vibration actuator has a larger amplitude response than any one of the plurality of linear resonant actuators for any given sinusoidal driving force frequency not equal to the resonant frequency of any one of the plurality of linear resonant actuators. In yet another example, the controller manages the virtual vibration actuator to implement a control sequence that undergoes optimal amplitude response immediately followed by optimal damping.

In accordance with another alternative, the controller manages the virtual vibration actuator to implement a control sequence that exhibits successive h-pulses of arbitrary amplitudes. In a further alternative, the virtual vibration actuator is controlled by the controller to implement a control sequence that undergoes optimal amplitude response followed by constant sinusoidal motion for N>1 half-wavelengths followed by optimal damping, wherein N is an integer value.

The virtual vibration actuator may be controlled by the controller to implement a control sequence that generates a sequence of h-pulses such that successive h-pulses differ in amplitude. In this case, changes in amplitude may have a relationship to at least one of changes in position of the vibration device and changes in orientation of the vibration device. In another example, the virtual vibration actuator is controlled by the controller to implement a control sequence that exhibits an h-pulse of arbitrary width. In this case, the arbitrary width may be determined by a resonant frequency associated with one or more of the plurality of linear resonant actuators. And for any of the above examples, the virtual vibration actuator may be controlled by the controller to exhibit superposition or sequential concatenation of multiple instances of the control sequence.

In accordance with another aspect of the invention, a vibration device is provided that comprises a mounting platform, a plurality of vibration actuators attached to the mounting platform, and a controller. Each of the plurality of vibration actuators having a moveable mass that is constrained to rotate about an axis of vibration and a spring attached to the moveable mass and the mounting platform. Each of the vibration actuators is configured to operate as a mechanical harmonic oscillator. The axes of vibration of the plurality of actuators are arranged in a substantially parallel configuration. The controller is coupled to each of the plurality of actuators and is configured to (1) control each actuator to impart a sinusoidal vibration torque of a frequency, f_1, onto the mounting platform, and (2) control amplitudes and phases of the sinusoidal vibration torque from each actuator to generate a combined vibration torque onto the mounting platform. The combined vibration torque emulates a single virtual vibration actuator having one or both of the following properties: a faster amplitude response than any one of the plurality of actuators, or a larger amplitude response than any one of the plurality of actuators for non-resonant frequencies.

In one example, the plurality of vibration actuators is a first plurality of actuators, and the vibration device further comprises a second plurality of vibration actuators attached to the mounting platform. Each of the second plurality of vibration actuators has a moveable mass that is constrained to rotate about an axis of vibration and a spring attached to the moveable mass and the mounting platform. Each of the second vibration actuators is configured to operate as a mechanical harmonic oscillator. The axes of vibration of the second plurality of actuators are arranged in a substantially parallel configuration with the first plurality of vibration actuators. The controller is coupled to each of the second plurality of actuators, and is configured to control each of the second plurality of actuators to impart the sinusoidal vibration torque of the frequency, f_1, onto the mounting platform such that movement of the moveable masses in the second plurality of vibration actuators counter-rotate relative to movement of the moveable masses in the first plurality of vibration actuators. The controller is also configured to control amplitudes and phases of the sinusoidal vibration torque from each actuator of the second plurality to generate a combined vibration force onto the mounting platform that emulates a single virtual vibration actuator.

For either of the above cases, all of the vibration actuators may be substantially identical. The vibration device may also be configured for haptic applications. In one example, the vibration device is further configured to be portable by a person. And in this case, the vibration device may be further configured to be worn or held by a person.

For any of the above examples, the vibration device may be further configured to provide haptic output for one of the following devices: a game controller, a motion game controller, a handheld game console, a remote control, a handheld portable computer, a navigation device, a handheld construction tool, a handheld surgical tool, a stylus, a plush toy, a pair of eyeglasses, a wristband, a wristwatch, a belt, an armband, a leg band, a mobile phone, a tablet computer, a device for aiding a vision-impaired person, a device for aiding a hearing-impaired person, and a device for augmenting reality with haptic feedback, a personal pleasure device for providing pleasurable haptic sensations, or a vibration device used for singly or in a pair for conveying telepresence.

In accordance with another aspect of the invention, a vibration device is provided that comprises a mounting platform, a plurality of linear resonant actuators attached to the mounting platform such that the axes of vibration for the actuators are substantially in a parallel configuration, and a controller. Each linear resonant actuator includes a moveable mass. The controller is coupled to each of the linear resonant actuators, and is configured to (1) control each one of the linear resonant actuators to impart a sinusoidal vibration force of a frequency, f_1, onto the mounting platform, and to (2) control amplitudes and phases of each sinusoidal vibration force to generate a combined vibration waveform onto the mounting platform. The combined vibration waveform emulates a virtual actuator having a maximum amplitude, A_1. The virtual vibration actuator has one or both of the following properties: a faster amplitude response than any one of the plurality of linear resonant actuators, or a larger amplitude response than any one of the plurality of linear resonant actuators for non-resonant frequencies.

In one example, the controller is configured to control the combined vibration waveform so that the virtual vibration actuator starts from rest and extends to the maximum amplitude A_1 in one quarter-wavelength of motion with no periods of reduced amplitude oscillation prior to maximal extension. In another example, the controller is configured to control the combined vibration waveform so that the virtual vibration actuator transitions from extension to the amplitude A_1 to a rest position in one quarter-wavelength of motion with no periods of reduced amplitude oscillation posterior to maximal extension. In a further example, the controller is configured to control the combined vibration waveform so that the virtual vibration actuator starts from rest, extends to a desired amplitude in one quarter-wavelength of motion and then returns to rest in the next successive quarter-wavelength of motion, with no periods of reduced amplitude oscillation at the onset or offset of the device's motion. Here, the virtual vibration actuator may be controlled by the controller to start from rest, extend to a selected amplitude in one quarter-wavelength of motion, oscillate about its rest position at the selected amplitude for an integer number of half-wavelengths, and then return to rest in one quarter-wavelength of motion, with no periods of reduced amplitude oscillation at the onset or offset of the vibration device's motion. For either of these examples, the controller may be configured to generate a successive series of the combined vibration waveforms, wherein each one of the waveforms in the successive series has the same amplitude. Alternatively, for these examples, the controller may be configured to generate a successive series of the combined vibration waveforms, wherein each one of the waveforms in the successive series has a different amplitude.

In accordance with another example, the virtual vibration actuator may have an oscillation period, T_1, which starts from rest, extends to a selected amplitude in one quarter-period of motion, and then returns to rest in the next successive quarter-period of motion, with no oscillations of reduced amplitude at the onset or offset of the vibration device's motion. In a further example, one or both of input forcing functions and output vibration patterns managed by the controller are related to either (1) an absolute spatial position and orientation of the vibration device, or (2) a spatial position and orientation of the vibration device relative to an external object, living entity or location. And in this case, one or both of input forcing functions and output vibration patterns managed by the controller may be dynamically determined by the spatial position and orientation of the vibration device relative to an external object, so that the vibration device is configured to guide a user towards or away from the external object, living entity or location.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a chart illustrating a number of different waveform types suitable for use with aspects of the present disclosure.

FIG. **2** illustrates a pair of vibration profiles having a phase difference.

FIG. **3** illustrates a pair of in-phase vibration profiles.

FIG. **4** illustrates a linear motion vibration actuator for use with aspects of the present disclosure.

FIGS. **5**A-B illustrate an example of a linear motion vibration actuator in accordance with aspects of the present disclosure.

FIGS. **6**A-B illustrate another example of a linear motion vibration actuator in accordance with aspects of the present disclosure.

FIGS. **7**A-B illustrate a further example of a linear motion vibration actuator in accordance with aspects of the present disclosure.

FIGS. **8**A-B illustrate yet another example of a linear motion vibration actuator in accordance with aspects of the present disclosure.

FIG. **9** illustrates a further example of a linear motion vibration actuator in accordance with aspects of the present disclosure.

FIG. **10** illustrates a vibration device in accordance with aspects of the present disclosure.

FIG. **11** illustrates the vibration device of FIG. **10** for generating a counterclockwise rotation in accordance with aspects of the present disclosure.

FIG. **12** illustrates the vibration device of FIG. **10** for generating a clockwise rotation in accordance with aspects of the present disclosure.

FIG. **13** illustrates the vibration device of FIG. **10** for generating a change in the direction of force in accordance with aspects of the present disclosure.

FIG. **14** illustrates a vibration device employing non-orthogonal linear actuators in accordance with aspects of the present disclosure.

FIG. **15** illustrates a vibration device employing a set of linear actuators for generation of a three dimensional force vector in accordance with aspects of the present disclosure.

FIG. **16** illustrates a game controller in accordance with aspects of the present disclosure.

FIG. **17** illustrates a vibration device in accordance with aspects of the present disclosure.

FIG. **18** illustrates another vibration device in accordance with aspects of the present disclosure.

FIG. **19** illustrates a vibration device for generating a combined torque in accordance with aspects of the present disclosure.

FIG. **20** illustrates another vibration device for generating a combined torque in accordance with aspects of the present disclosure.

FIG. **21** illustrates a rotary vibration actuator with eccentric mass in accordance with aspects of the present disclosure.

FIG. **22** illustrates a vibration device with a pair of eccentric mass actuators in accordance with aspects of the present disclosure.

FIG. **23** illustrates synchronous vibration of eccentric mass actuators in accordance with aspects of the present disclosure.

FIGS. **24**A-C illustrate a pivoting actuator in accordance with aspects of the present disclosure.

FIGS. **25**A-C illustrate another pivoting actuator in accordance with aspects of the present disclosure.

FIG. **26** illustrates a pivoting actuator utilizing a pair of spring devices in accordance with aspects of the present disclosure.

FIGS. **27**A-F illustrate a further pivoting actuator in accordance with aspects of the present disclosure.

FIG. **28** illustrates a synchronized vibration system employing rotary actuators in accordance with aspects of the present disclosure.

FIGS. **29**A-B illustrate game controllers in accordance with aspects of the present disclosure.

FIG. **30** illustrates a rocking actuator in accordance with aspects of the present disclosure.

FIG. **31** illustrates a vibration system in accordance with aspects of the present disclosure.

FIG. **32** illustrates control of a vibration system in accordance with aspects of the present disclosure.

FIG. **33** illustrates control of a vibration system in accordance with aspects of the present disclosure.

FIG. **34** illustrates control of a vibration system in accordance with aspects of the present disclosure.

FIG. **35** illustrates a vibration system in accordance with aspects of the present disclosure.

FIGS. **36**A-B illustrate equation parameter and pattern selection processing in accordance with aspects of the present disclosure.

FIG. **37** illustrates a haptic interface system in accordance with aspects of the present disclosure.

FIG. **38** illustrates another haptic interface system in accordance with aspects of the present disclosure.

FIG. **39** illustrates control of vibration profiles in accordance with aspects of the present disclosure.

FIG. **40** illustrates a vibration actuator in accordance with aspects of the present disclosure.

FIG. **41** illustrates another vibration actuator in accordance with aspects of the present disclosure.

FIG. **42** illustrates a vibration device controller in accordance with aspects of the present disclosure.

FIG. **43** illustrates a vibration device with two linear resonant actuators for use with aspects of the disclosure.

FIG. **44** illustrates superposition of two synchronized sine waves with a phase offset that generates a combined waveform with asymmetry according to aspects of the disclosure.

FIG. **45** illustrates time steps within a vibration cycle of two linear resonant actuators generating an asymmetric waveform according to aspects of the disclosure.

FIG. **46** illustrates two linear resonant actuators directly attached to one another for use with aspects of the disclosure.

FIG. **47** illustrates an alternative example of two linear resonant actuators attached in line with one another for use with aspects of the disclosure.

FIG. **48** illustrates a vibration device that uses a slider-crank linkage for use with aspects of the disclosure.

FIG. **49** illustrates a vibration device with n LRAs for use with aspects of the disclosure.

FIG. **50** illustrates an asymmetric pulse train according to aspects of the disclosure.

FIG. **51** illustrates a pulse train with zero DC according to aspects of the disclosure.

FIG. **52** is a flow diagram illustrating a process for maximizing asymmetry according to aspects of the disclosure.

FIG. **53** illustrates an example of waveform asymmetry according to aspects of the disclosure.

FIG. **54** illustrates another example of waveform asymmetry according to aspects of the disclosure.

FIG. **55** illustrates a further example of waveform asymmetry according to aspects of the disclosure.

FIG. **56** illustrates synchronized triangular waveforms according to aspects of the disclosure.

FIG. **57** illustrates a vibration device that can generate asymmetric torques according to aspects of the disclosure.

FIG. **58** illustrates a controller for General Synchronized Vibration of a pair of linear force actuators according to aspects of the disclosure.

FIG. **59** illustrates a linear force actuator with a sensor that detects when a moving mass passes a midpoint position according to aspects of the disclosure.

FIG. **60** illustrates a sensor attached to a mounting platform according to aspects of the disclosure.

FIG. **61** illustrates a vibration device controller that uses sensor measurements to update a commanded amplitude, phase and/or frequency according to aspects of the disclosure.

FIG. **62** illustrates a vibration device that includes two orthogonal sets of LRAs according to aspects of the disclosure.

FIG. **63** illustrates a vibration device that includes two non-orthogonal sets of LRAs according to aspects of the disclosure.

FIG. **64** illustrates an ERM for use with aspects of the disclosure.

FIG. 65 illustrates a vibration device using an arbitrary number of ERMs according to aspects of the disclosure.

FIG. 66 illustrates a vibration device having 4 ERMs for use with aspects of the disclosure.

FIG. 67 illustrates time steps within a vibration cycle of ERMs generating an asymmetric waveform according to aspects of the disclosure.

FIG. 68 illustrates an example vibration device with a plurality of ERM pairs.

FIG. 69 illustrates a vibration device with four vertically stacked ERMs in one example used according to aspects of the disclosure.

FIG. 70 illustrates time steps of an asymmetric waveform for a vibration device with four ERMs that are vertically stacked, according to aspects of the disclosure.

FIG. 71 illustrates a vibration device with two ERMs that rotate in the same direction.

FIG. 72 illustrates a vibration device with four co-rotating pairs of ERMs according to aspects of the disclosure.

FIGS. 73A-B illustrate vibration devices with two ERMs mounted in different arrangements according to aspects of the disclosure.

FIG. 74 illustrates an eccentric mass configured for use as a reaction wheel according to aspects of the disclosure.

FIG. 75 illustrates an ERM pair with interleaved masses according to aspects of the disclosure.

FIGS. 76A-B illustrate example configurations having three ERMs for use with aspects of the disclosure.

FIG. 77 illustrates another configuration with three ERMs arranged in a row.

FIG. 78 illustrates an ERM with a sensor for use with aspects of the disclosure.

FIG. 79 illustrates an ERM with a reflective optical sensor for use with aspects of the disclosure.

FIG. 80 illustrates an ERM with a line of sight sensor for use with aspects of the disclosure.

FIG. 81 illustrates an ERM with a Hall effect sensor for use with aspects of the disclosure.

FIG. 82 illustrates a vibration device with four ERMs arranged in a row for use with aspects of the disclosure.

FIG. 83 illustrates time steps of a waveform with cancellation of forces according to aspects of the disclosure.

FIG. 84 illustrates a vibration device with two pairs of ERMs that share the same center.

FIGS. 85A-B illustrate an ERM pair with interleaved masses having varying thickness according to aspects of the disclosure.

FIGS. 86A-C illustrate an ERM pair with interleaved masses having support bearing according to aspects of the disclosure.

FIG. 87 illustrates haptic feedback within a system having a visual display according to aspects of the disclosure.

FIG. 88 illustrates another example of haptic feedback within a system having a visual display according to aspects of the disclosure.

FIG. 89 illustrates a vibration device with sensor feedback according to aspects of the disclosure.

FIG. 90 illustrates a locomotion device for use with aspects of the disclosure.

FIG. 91 is a diagram illustrating six dimensions of a Synchronized Array of Vibration Actuators in a Network Topology ("SAVANT") Control Space in accordance with aspects of the present disclosure.

FIG. 92A illustrates an example of a SAVANT node having a single LRA in accordance with aspects of the present disclosure.

FIG. 92B illustrates an example of a SAVANT node having two LRAs arranged in a stack in accordance with aspects of the present disclosure.

FIG. 92C illustrates an example of a SAVANT node having three LRAs arranged in a stack in accordance with aspects of the present disclosure.

FIG. 92D illustrates an example of a SAVANT node having two LRAs in a compact planar arrangement in accordance with aspects of the present disclosure.

FIG. 92E illustrates an example of a SAVANT node having three LRAs in a compact planar arrangement in accordance with aspects of the present disclosure.

FIG. 92F illustrates an example of a SAVANT node having three LRAs in a compact arrangement with their axes of vibration spanning three dimensions in accordance with aspects of the present disclosure.

FIG. 92G illustrates an example of a SAVANT node having three LRAs in a cube arrangement with their axes of vibration spanning three dimensions in accordance with aspects of the present disclosure.

FIG. 92H illustrates an example of a SAVANT node having six LRAs in a cube arrangement with their axes of vibration spanning three dimensions in accordance with aspects of the present disclosure.

FIG. 92I illustrates an example of a SAVANT node having twelve LRAs in a cube arrangement with their axes of vibration spanning three dimensions in accordance with aspects of the present disclosure.

FIG. 92J illustrates an example of a SAVANT node having four LRAs in a tetrahedral arrangement with their axes of vibration spanning three dimensions in accordance with aspects of the present disclosure.

FIG. 93 illustrates an example of a SAVANT node having three LRAs in a triangular planar arrangement in accordance with aspects of the present disclosure.

FIG. 94 illustrates an example portable client device incorporating a SAVANT node in accordance with aspects of the present disclosure.

FIG. 95 illustrates another example portable client device incorporating a plurality of SAVANT nodes in accordance with aspects of the present disclosure.

FIG. 96 illustrates an example of a handheld game controller incorporating one or more SAVANT nodes in accordance with aspects of the present disclosure.

FIG. 97 illustrates an h-pulse of a 3-LRA system at an initial point in time in accordance with aspects of the present disclosure.

FIG. 98 illustrates an h-pulse of the 3-LRA system at a second point in time in accordance with aspects of the present disclosure.

FIG. 99 illustrates an h-pulse of the 3-LRA system at a third point in time in accordance with aspects of the present disclosure.

FIG. 100 illustrates an h-pulse of the 3-LRA system at a fourth point in time in accordance with aspects of the present disclosure.

FIG. 101 is a chart of an example oscilloscope trace from the measurement of the ramp-up of an LRA from rest, when driven with a 150 Hz input signal.

FIG. 102 is a graph of a model fitted to 150 Hz LRA data in accordance with aspects of the present disclosure.

FIG. 103 is a graph of the response times for a 2-LRA system compared with a 1-LRA system in accordance with aspects of the present disclosure.

FIG. 104 is a graph of the response times for a 3-LRA system compared with the 2-LRA and 1-LRA systems in accordance with aspects of the present disclosure.

FIG. **105** is a graph of the frequency response curves for various systems of 150 Hz LRAs in accordance with aspects of the present disclosure.

FIG. **106** represents four snapshot views of the resultant motion for two orthogonal springs driven with the same amplitude and phase in accordance with aspects of the present disclosure.

FIG. **107**A represents four snapshot views of the resultant motion for two orthogonal springs driven with the same amplitude but out-of-phase by 180° in accordance with aspects of the present disclosure.

FIG. **107**B represents four snapshot views of the resultant motion for two orthogonal springs producing a linear vibrational effect with a time-varying direction in accordance with aspects of the present disclosure.

FIG. **108**A represents four snapshot views of the resultant motion for two orthogonal springs driven with the same amplitude but out-of-phase by 90° in accordance with aspects of the present disclosure.

FIG. **108**B represents four snapshot views of the resultant motion for two orthogonal springs producing a circular vibrational effect with time-varying radius in accordance with aspects of the present disclosure.

FIG. **109** represents four snapshot views of the resultant motion for two orthogonal springs driven with different amplitudes and out-of-phase by 90° in accordance with aspects of the present disclosure.

FIG. **110**A represents four snapshot views of the resultant motion for two orthogonal springs driven with the different amplitudes and out-of-phase by 22.5° in accordance with aspects of the present disclosure.

FIG. **110**B represents four snapshot views of the resultant motion for two orthogonal springs producing an elliptical vibrational effect with time-varying direction and axes in accordance with aspects of the present disclosure.

FIG. **111** is a graph of an example Lissajous curve produced by two orthogonal LRAs in accordance with aspects of the present disclosure.

FIG. **112** is a plot of the beat pattern produced by two parallel LRAs driven at 175 Hz and 180 Hz respectively in accordance with aspects of the present disclosure.

FIG. **113** is a plot of the Sawtooth wave approximation in a 5-LRA system. The amplitudes and driving frequencies of the LRAs are given by the Fourier series approximation of $f(t)=t$ in accordance with aspects of the present disclosure.

FIG. **114** is a plot of an Asymmetric waveform produced by three 2-LRA systems driven at the first three harmonics of 22.5 Hz in accordance with aspects of the present disclosure.

FIG. **115** is a plot of an Asymmetric waveform produced by three 2-LRA systems driven at the first three harmonics of 4.5 Hz in accordance with aspects of the present disclosure.

FIG. **116** is a plot of an h-pulse control effect for a 3-LRA SAVANT in accordance with aspects of the present disclosure.

FIG. **117** is a plot of a Scaled h-pulse of a 3-LRA SAVANT in accordance with aspects of the present disclosure.

FIG. **118** is a plot of an Amplitude Seeking control effect using a 3-LRA system in accordance with aspects of the present disclosure.

FIG. **119** is an example of a Vibrational Gradient produced with a 3-LRA system in accordance with aspects of the present disclosure.

FIG. **120** is an example of an Elongated h-pulse created by 150 Hz LRAs driven at 25 Hz in accordance with aspects of the present disclosure.

FIG. **121** is a graph of the response times for 3-LRA, 2-LRA and 1-LRA systems modeled with a resonant frequency of 175 Hz in accordance with aspects of the present disclosure.

FIG. **122** is a graph illustrating Optimal Braking of a 3-LRA System in accordance with aspects of the present disclosure.

DETAILED DESCRIPTION

The foregoing aspects, features and advantages of the present disclosure will be further appreciated when considered with reference to the following description of preferred embodiments and accompanying drawings, wherein like reference numerals represent like elements.

As used herein, an actuator is a device that can generate mechanical motion or force. Actuators can convert a source of energy into mechanical motion or force. The source of energy can be electrical, pneumatic, hydraulic, or another source. Examples of actuators include rotary and linear motors. Examples of electric actuators include DC, AC, and stepper motors.

The term "direction" includes the orientation of an axis, also referred to as vector direction. A vector aligned with a specific direction can be either in the positive direction along the axis or the negative direction along the axis. As used herein, the term direction may distinguish between all angles in a circle, such as 0 to 360 degrees. And vibration control may distinguish between positive and negative directions along a single axis. Furthermore, the term "controller" is used herein in some situations to reference to game controller, and in other situations to a real-time controller of actuators, such as a microprocessor or an ASIC.

In this disclosure, the term "General Synchronized Vibration" refers to control of the timing, and in some cases also control of amplitude, of multiple vibration forces, torques, or forces and torques. The sources of these vibration forces and torques can be electromagnetic, electrostatic, magnetic, spring forces, inertial forces such as centripetal forces, piezoelectric, pneumatic, hydraulic, or other force and torque sources. The sources of these vibration forces and torques can include those described in the text "Engineering Haptic Devices: A Beginner's Guide for Engineers" by Thorsten A. Kern, © 2009 (the entire disclosure of which is hereby expressly incorporated by reference herein). These vibration forces and torques can be generated from separate Vibration Actuators or from actuators that generate multiple force, torques, or forces and torques. In General Synchronized Vibration the forces, torques, or forces and torques are vectorially combined so that they generate a combined force, torque, or force and torque onto an object. The vector combination of force and torque vectors is also referred to as superposition. General Synchronized Vibration results in a combined vibration force, a combined vibration torque, or a combined vibration force and vibration torque onto an object. A force applied onto an object can also apply a torque onto that object. Accordingly, the references in this document to force also apply to force and torque unless explicitly described otherwise.

In the event that there is a difference in the usage of terminology between the instant application and any wholly included reference identified herein, the usage of the differing term definitions will be governed by the use in the present disclosure.

A vibration (or vibratory) actuator can impart repeated forces onto an object. These repeated forces can repeat a similar force profile over time during each repetition. Examples include rotary motors with eccentric masses, and linear actuators which move masses back and forth. These actuators can be DC, AC, stepper, or other types of actuators. A vibration actuator can repeat a similar force profile (waveform) in each cycle, or there can be variations in force profiles between cycles. Variations between cycles can be in amplitude, frequency, phase, and profile shape.

When a force is generated in a repeated cycle it can generate a vibratory force. The profile (also referred to as a waveform) of a repeated force cycle can be in a sinusoidal shape, triangular wave, a square wave, or other repeated profile as shown in FIG. **1**. The frequency of vibration describes how frequently a vibration cycle is repeated. A frequency of vibration, f, is defined as the number of vibrations per unit time, and often is given in Hertz whose units are cycles per second. The period of vibration, T, is the duration of each cycle in units of time. The mathematical relationship between frequency and period of vibration is given by the following equation:

$$f=1/T \tag{1}$$

A vibration force, F, is in a repeated cycle when

$$F(t+T)=F(t) \tag{2}$$

where T is the period of vibration and t is time.

For purposes of vibration devices it is sufficient for the period of vibration to be approximate, and therefore a vibration is considered to be in a repeated cycle when:

$$F(t+T)\approx F(t) \tag{3}$$

One vibration waveform is a sinusoidal waveform, where the vibration force can be given by:

$$F(t)=A\sin(\omega t+\phi) \tag{4}$$

Here, F(t) is force as a function of time. A is the maximum amplitude of force. ω is the frequency of vibration in radians per second (the frequency in Hertz is $f=\omega/(2\pi)$). And ϕ is the phase of vibration in radians. When $\omega t=2\pi$ the force profile repeats itself.

A vibration actuator may impart repeated forces onto an object. Due to the dynamics of an actuator, a single actuator can impart forces at multiple frequencies at the same time. However, for the purposes of analyzing vibrations and describing vibration devices herein, the primary frequency of an actuator's motion means the frequency having the largest component of kinetic energy in it.

The period of vibration can be defined by the time elapsed between the beginning of one vibration cycle and beginning of the next cycle. Thus to identify the period of vibration it is useful to identify the beginning of a cycle. One method for defining the beginning of cycle is to define the beginning of the cycle as the point with maximum amplitude in the profile. FIG. **1** is an amplitude versus time chart **10** showing the vibration profiles of a sine wave **12**, a triangle wave **14**, an arbitrarily shaped profile **16**, and a square wave **18**. The period for each of these profiles is designated by T.

The sine wave **12**, triangle wave **14**, and arbitrary profile wave **16** all have a unique point of maximum amplitude during each repeated cycle, and this point of maximum amplitude is used to define the beginning of the cycle. The square wave **18** does not have a unique point of maximum amplitude within a cycle; in such cases a repeated point on the profile can be selected to designate the beginning of the cycle. In FIG. **1**, the point at which the square wave **18** transitions from a low value to a high value is designated at the beginning point of the cycle, and used use to define the period of the repeated profile. Thus, any profile that can be represented as repeated cycles can represent a vibration.

A frequency of vibration can also be identified when the shape of signal does not consist of exactly repeated profiles. Variations in amplitude of the cycle and small changes in the shape of a cycles profile still allow one to identify a unique point that designates the beginning of the cycle. As long as a repeated point in the profile can be identified, then the beginning of each cycle, a vibration period, and vibration frequency can be determined.

The phase of vibration defines the timing of the beginning of a cycle of vibration. A phase difference between two vibration waveforms is defined as the difference between the beginning of a vibration cycle in one waveform and the beginning of a vibration cycle in the other waveform. If there is a nonzero difference in the phase of vibration between two profiles, then the beginning of the cycles do not coincide in time. FIG. **2** is an amplitude versus time chart **20** showing two vibration profiles, **22** and **24**, with a phase difference Δ between them. The phase difference Δ can be given in units of time, such as shown in FIG. **2**. Alternatively, the phase of vibration can also be given in radians for sinusoidal vibrations. When the phase difference Δ between two waveforms is zero, then the two waveforms are considered to be in-phase, as shown in the amplitude versus time chart **30** of FIG. **3**.

As long as it is possible to identify the beginning of a cycle it is possible to identify a phase of vibration, even when the amplitude and frequency of vibration change between cycles of vibration.

One implementation of synchronized vibration is a vibration force formed by the superposition of two or more vibration waveforms where each of the waveforms include peaks that coincide in time with the peaks of the other waveforms on a regularly repeating basis. In a preferred embodiment, each of the waveforms would have the same frequency and a specified phase difference between them. Superposition can preferably be the vector sum of forces, torque, or forces and torque. Typically, the sources of these vibration waveforms are different vibration actuators. Often in synchronous vibration the waveforms have a zero phase difference between them, and thus the vibration waveforms are in-phase and in synchronous vibration. As used herein, specified phase difference may range between and including 0° and 360°. In some embodiments, the specified phase difference is 0° or 180°. In synchronized vibration, the various vibration waveforms can have different amplitudes. FIG. **3** illustrates two vibration waveforms of triangular profile that are synchronized. Both of these waveforms have the same frequency, they have different amplitudes, and the waveforms are in-phase. The maximum amplitude of both waveforms in FIG. **3** occurs at the same time.

Typically, synchronized vibration profiles will have similar shaped profiles. However, vibration actuators with different shaped vibration profiles can also be vibrated synchronously by matching frequency of vibration and specifying the phase difference between the waveforms. The matching of phase and frequency of vibration can be done approximately and still result in synchronized vibration.

Synchronized vibration can be generated by adding two vibration profiles together, where the amplitude of the second vibration profile is a multiple of the amplitude of the first vibration profile. This multiplying factor can be either positive or negative.

If there are two or more vibrating actuators where the peak amplitude of force of each vibrating actuator occurs repeatedly at approximately the same time, then these actuators are in-phase and in synchronous vibration. The peak amplitude of force can be either in the positive or negative direction of the vibration actuators' or vibration device's coordinate system. Thus if a positive peak amplitude from one actuator occurs at approximately the same time as the negative peak amplitude of another actuator, then these actuators are in-phase and are in synchronous vibration.

An exemplary linear motion vibration actuator **100** is shown in FIG. **4**. As shown, the linear motion vibration actuator **100** contains a moving mass **102** and a base **104**. The moving mass **102** moves relative to the base **104** in a back and forth linear motion. Force can be applied from the base **104** to the moving mass **102** and in a similar fashion from the moving mass **102** onto the base **104**. The force transfer can occur, for instance, via magnetic forces, spring forces, and/or lead screw forces. Examples of linear actuators suitable for use in accordance with the present disclosure are described in U.S. Pat. Nos. 5,136,194 and 6,236,125, and in U.S. patent application Ser. No. 11/325,036, entitled "Vibration Device," the entire disclosures of which are hereby incorporated by reference herein.

As the moving mass **102** in the linear motion vibration actuator **100** moves back and forth, forces are generated between the moving mass **102** and the base **104**. These forces can be transmitted through the base **104** of the actuator **100** to an object that the actuator is mounted to (not shown). The moving mass **102** may also be attached to an object, such as a handle (not shown), that is external to the actuator **100**, and may transmit forces directly to an object external to the actuator **100**.

The forces in the linear motion vibration actuator **100** may be magnetic forces, such as with a voice coil. The moving mass **102** may contain, for instance, a permanent magnet, electromagnet, ferromagnetic material, or any combination of these. The base **104** may contain, for instance, a permanent magnet, an electromagnet, ferromagnetic material, or any combination of these. Magnetic forces may be generated between base **104** and the moving magnet that generate acceleration and motion of the moving mass **104**. A force in the linear motion vibration actuator **100** generated with an electromagnet can be modulated by controlling the current flowing through the electromagnet.

One embodiment of linear motion vibration actuator **100** in accordance with the present disclosure is shown in FIGS. **5**A-B as linear motion vibration actuator **110**. Actuator **110** preferably contains a moving mass **112** that comprises an electromagnet, as well as a permanent magnet **116** attached to the base **114**. The motion of the moving mass **112** is along the x axis as shown in the side view in FIG. **5**A. The magnetization polarity of the permanent magnet **116** is along the x axis as shown by the North and South poles on the permanent magnet **116**. The electromagnet is preferably configured as a coil wound about the x axis. As shown in the end view of FIG. **5**B, in the present embodiment the shape of the electromagnet is desirably cylindrical and the shape of the permanent magnet **116** is desirably tubular, although the electromagnet and the permanent magnet **116** may have any other configuration. In this embodiment both the electromagnet and the permanent magnet **116** may have ferromagnetic material placed adjacent to them to increase the force output of the actuator **110**.

In this embodiment, the force in the actuator **110** can be modulated by controlling the current in the electromagnet. When the current in the electromagnet flows in one direction, then the magnetic force will push the moving mass **112** towards one side of the actuator. Conversely when the current in the electromagnet flows in the other direction, then the moving mass **112** will be pushed to the other side of the actuator **110**. Increasing the amount of current in the electromagnet will increase the amount of force applied onto the moving mass **112**.

Another embodiment of the linear motion vibration actuator **100** in accordance with the present disclosure is shown in FIGS. **6**A-B. Here, linear motion vibration actuator **120** preferably contains a moving mass **122** that comprises a permanent magnet, as well as an electromagnet magnet **126** attached to base **124**. The motion of the moving mass **122** is along the x axis as shown in the side view in FIG. **6**A. The magnetization polarity of the permanent magnet is along the x axis as shown by the North and South poles on the permanent magnet. The electromagnet **126** is preferably a coil wound about the x axis. As shown in the end view of FIG. **6**B, in this embodiment the shape of the electromagnet **124** is tubular and the shape of the permanent magnet is cylindrical.

In this embodiment both the electromagnet **124** and the permanent magnet of the moving mass **122** may have ferromagnetic material placed adjacent to them to increase the force output of the actuator **120**. The force in the actuator **120** can be modulated by controlling the current in the electromagnet **124**. When the current in the electromagnet **124** flows in one direction, then the magnetic force will push the moving mass **122** towards one side of the actuator **120**. Conversely when the current in the electromagnet flows in the other direction, then the moving mass **122** will be pushed to the other side of the actuator **120**. Increasing the amount of current in the electromagnet will increase the amount of force applied onto the moving mass **122**.

Another embodiment of the linear motion vibration actuator **100** in accordance with aspects of the present disclosure is shown in FIGS. **7**A-B, which is similar to the embodiment shown in FIGS. **6**A-B. Here, actuator **130** includes a moving mass **132** and a base **134**. The moving mass **132** preferably comprises a permanent magnet. An electromagnet **136** at least partly surrounds the moving mass **132**. The electromagnet **136** is desirably connected to the base **134**. Unlike the actuator **120**, the actuator **130** in this embodiment preferably includes one or more springs **138** that are attached to the base **134** and to the moving magnet **132** at either end, as shown in the side view of FIG. **7**A. The springs **138** are operable to generate forces in a direction that returns the moving mass **132** to a center position, for instance midway between either end of the electromagnet **136**.

The springs **138** function to keep the moving mass **132** close to the center position when the actuator power is off, and to provide a restoring force when the moving mass **132** is at one end of travel of the actuator **130**. The stiffness of the springs **138** can be selected so that the natural frequency of the actuator **130** increases the amplitude of vibration at desired natural frequencies. This spring effect can be generated from a single spring, from a nonlinear spring, from extension springs, as well as compression springs. A number of such spring configurations which may be employed with the present disclosure are described in the aforementioned U.S. patent application Ser. No. 11/325,036.

Another embodiment of the linear motion vibration actuator **100** according to aspects of the present disclosure is shown in FIGS. **8**A-B. This embodiment is similar to the embodiments shown in FIGS. **6**A-B and **7**-B in that actuator **140** includes a moving mass **142** including a permanent magnet, a base **144**, and an electromagnet **146** coupled to the base **144** and at least partly surrounding the moving mass **142**. The electromagnet **146** may be, e.g., rigidly or semi-rigidly coupled such that a vibration force is transmitted from the actuator **140** to the base **144**, for instance to enable a user to perceive the vibration force. In this embodiment, a pair of permanent magnets **148** is attached to the base and are in operative relation to the moving magnet **142** at either end as shown in the side view of FIG. **8**A. The permanent magnets **148** have poles, as shown by the N and S in FIG. **8**A, which are configured to repel the moving mass **142** and to generate forces in a direction that returns the moving mass **142** to a center position. The permanent magnets **148** function to keep the moving mass **142** close to a center position when the actuator power is off, and to provide a restoring force when the moving mass **142** is at one end of travel of the actuator **140**.

The size of the permanent magnets **148** attached to the base **144** can be selected so that the natural frequency of the actuator **140** increases the amplitude of vibration at desired natural frequencies. The actuator **140** may be controlled so that one or more natural frequencies are selected during different modes or times of operation. Use of repulsive magnetic forces as shown in FIG. **8**A to generate centering forces on the moving permanent magnet of the moving mass **142** can provide lower friction than use of springs **138** as shown in FIG. **7**A, and thus can generate increased actuator efficiency and smoothness. A number of configurations showing use of permanent magnets to center a moving mass, which are suitable for use in the present disclosure, are described in the aforementioned "Vibration Device" patent application.

Alternative embodiments of linear motion vibration actuators that may also be utilized with the present disclosure include both springs and magnets, either alone or in combination, that return a moving mass towards the center of range of motion of the actuator.

A further alternative embodiment of the linear motion vibration actuator **100** in accordance with the present disclosure is shown in FIG. **9**. This embodiment comprises actuator **150**, which is similar to a solenoid in that it has a ferromagnetic moving plunger **152** for moving relative to a base **154**. The plunger **152** is pulled into an electromagnetic coil **156** when current flows through the coil **156**. The coil **156** is coupled to the base **154**. A ferromagnetic end piece **158** can be located within or at the end of the coil **156** to increase the force output of the actuator **150**. A spring device **160** may be positioned opposite the end piece **158**. The spring device **160** is preferably employed to retract the plunger **152** out of the coil **156**. As shown in FIG. **9**, both an end of the coil **156** and an end of the spring **160** are desirably fixed to the base **154** of the actuator **150**. The coil **156** and the spring **160** may be fixed to a single base at different sections thereon, or may be fixed to separate base elements that are coupled together. The current in the coil **156** can be turned on and off to generate a vibration force.

A preferred embodiment of a vibration device **200** according to the present disclosure is shown in FIG. **10**. In this embodiment, the vibration device **200** preferably includes two linear motion vibration actuators mounted on to it, namely actuator **202** and actuator **204**. The actuator **202** includes a moving mass **206** and the actuator **204** includes a moving mass **208**. The vibration actuators **202**, **204** are attached to the vibration device **200** in a manner that transmits the force from the vibration actuators **202**, **204** to the vibration device **200**. Preferably the vibration device **200** has an enclosure or base (not shown) to which the vibration actuators **202**, **204** are connected.

The vibration actuators **202**, **204** are desirably attached in a relatively rigid fashion to the vibration device enclosure or base. Rigid attachment provides a common base to the vibration device **200**, upon which forces from both vibration actuators **202**, **204** are applied. In this embodiment, the two actuators **202**, **204** are mounted at approximately right angles to each other. The force generated by actuator **202** is shown as force vector F1, and the force vector from actuator **204** is shown as F2. As expressed herein, vectors and matrices are designated by bold font and scalars are designated without bolding. The combined force generated by the vibration device **200** is the vector sum of the vibration forces from both of the actuators **202**, **204**, and is shown in FIG. **10** as vector Fcombined.

The combined force, Fcombined, applied by the vibration actuators **202** and **204** onto the vibration device **200** is a superposition of the vibration forces from each actuator, and is a function of time, t. The force vector can Fcombined(t) is given by the vector equation:

$$F_{combined}(t)=F_1(t)+F_2(t) \tag{5}$$

where $F_1(t)$ is the force vector from actuator **202** as a function of time, and $F_2(t)$ is the force vector from actuator **204** as a function of time.

Both actuators **202**, **204** can be operated in a vibratory fashion. For the case of a sine wave vibration, the actuator forces can be given by:

$$F_1(t)=a_1A_1 \sin(\omega_1 t+\phi_2) \tag{6}$$

and

$$F_2(t)=a_2A_2 \sin(\omega_2 t+\phi_2) \tag{7}$$

respectively, where A_1 and A_2 are the respective amplitudes of vibration, a_1 and a_2 are the unit vectors corresponding to the respective directions of vibration, ω_1 and ω_2 are the respective frequencies of vibration, ϕ_2 and ϕ_2 are the respective phase angles, and t is time. Other profile vibrations including square waves, triangle waves, and other profiles can also be implemented with each actuator.

In the example shown in FIG. **10**, actuator **202** is aligned with the y axis, and thus the unit vector a_1 is represented by:

$$a_1=\begin{bmatrix}0\\1\end{bmatrix} \tag{8}$$

and the unit vector a_2 aligned with the x axis and is represented by:

$$a_2=\begin{bmatrix}1\\0\end{bmatrix} \tag{9}$$

The combined force vector, $F_{combined}$, is given by the superposition of forces form the actuators **202** and **204**, and thus is given by:

$$F_{combined}(t)=a_1A_1 \sin(\omega_1 t+\phi_2)+a_2A_2 \sin(\omega_2 t+\phi_2) \tag{10}$$

It is possible to vibrate actuators **202** and **204** shown in FIG. **10** in a manner that is in-phase and in synchronous vibration. Under such vibration, there will be a single vibration frequency, ω and a single phase ϕ Accordingly, $F_{combined}$ can be given by:

$$F_{combined}(t)=[a_1A_1+a_2A_2] \sin(\omega t+\phi) \tag{11}$$

With such in-phase and synchronous vibration the vibration is synchronized, then the peak forces from both linear motion vibration actuators will occur at the same instances during each cycle of vibration. The net direction of vibration force is the vector combination of $[a_1A_1+a_2A_2]$. Thus, in synchronized vibration and in-phase vibration, the vibration device generates a vibration force at a specified frequency in a specified direction that results from the vector combination of forces from the direction and magnitude of each of the actuators in the device. It is possible to control the magnitude of vibration in each linear motion vibration actuator, and thereby control the net direction of vibration of $F_{combined}$.

In a preferred example, the vibration frequency, ω, phase ϕ, and waveform of each actuator are substantially identical. For instance, ω_2 may be set to be substantially equal to ω_1 and ϕ_2 may be set to be substantially equal to ϕ_1. By way of example only, ω_2 may be set to within 10% of the value of ω_1, more preferably to within 5% of the value of ω_1. Similarly, by way of example only, ϕ_2 may be set to within 10% of the value of ω_1, more preferably to within 5% of the value of ϕ_1. In another example, the frequencies and/or phases may be set exactly equal to one another. Alternatively, the frequencies, phases, and/or waveforms of each actuator may be set so that a user would not be able to notice the difference in frequency, phase or waveform. In a further alternative, if the vibration device is used in a haptic application to generate force sensations on the user, small variations may occur which may not be detected by the user or which cannot be significantly felt by the user. In other instances, force sensations in a haptic application or in a vibratory feeder application may vary minutely so that user performance in the haptic application or performance of the vibratory feeder is not significantly changed.

It is also possible to apply equation 11 to a vibration profile/waveform of arbitrary shape. Here, waveform p(t) may be used to represent the waveform shape over time t. A period of vibration may be represented by p(t)=p(t+nT), where n=1, 2, 3, etc. and T is the period of vibration. In this case, an arbitrarily shaped synchronized vibration profile may be represented as:

$$F_{combined}(t)=[a_1(t)A_1(t)+a_2(t)A_2(t)]p(t) \tag{11.1}$$

When the direction of vibration force for each actuator is substantially constant relative to a base member, the arbitrarily shaped synchronized vibration profile may be represented as:

$$F_{combined}(t)=[a_1A_1(t)+a_2A_2(t)]p(t) \tag{11.2}$$

To illustrate how the direction of $F_{combined}$ can be controlled, the peak magnitudes, A_1 and A_2, are represented in FIGS. **10** and **11** by the location of the moving masses **206** and **208** within each of the actuators **202** and **204**, respectively. In FIG. **10**, both actuator **202** and actuator **204** are desirably vibrated at the same amplitude, and the corresponding $F_{combined}$ is at approximately a 45 degree angle between the actuators **202**, **204**.

By varying the magnitude of the vibration force in the actuators **202**, **204**, it becomes possible to control the direction of vibration of the combined force effect. In FIG. **11**, the actuator **202** is vibrating at peak amplitude as illustrated by the peak position of moving mass **206** at the end of travel limits of actuator **202**. However, actuator **204** is vibrating at a lower peak amplitude, as illustrated by the peak position of moving mass **208** closer to the middle of travel limits of actuator **204**. The lower peak force is also illustrated in FIG. **11** by the shorter length vector for F_2. The direction of the combined force, $F_{combined}$, is the result of vector addition of F_1 and F_2, and for vibrations illustrated in FIG. **11** is rotated counterclockwise relative to the direction shown in FIG. **10**.

In a similar fashion, the direction of combined force can be rotated in the clockwise direction as shown in FIG. **12**. The vibration case illustrated in FIG. **12** shows the peak amplitude of vibration of actuator **202** reduced relative to that shown in FIG. **10**, while the peak amplitude of actuator **204** remains high. In this case, the vector addition of F_1 and F_2 results in a clockwise rotation of $F_{combined}$ in FIG. **12** relative to the direction shown in FIG. **10**.

It is also possible to change the direction of $F_{combined}$ to an adjacent quadrant. As shown in FIG. **13**, the sign of the F_2 has changed be in the direction of the negative x axis, relative to the positive x direction that shown in FIG. **10**. The change in sign of F_2 can be achieved by changing the sign of A_2 in equation 11 above. It should be noted that one could achieve a similar representation of the combined force equation by defining actuator **204** vibration as at 180 degrees out of phase of actuator **202**. However, changing the sign on the actuators vibration amplitude maintains the form of equation of synchronous vibration shown in equation 11. Thus, vibration that can be represented as 180 degrees out of phase can also be represented as in-phase vibration but with a negative amplitude of vibration.

An alternative embodiment of a vibration device in accordance with the present disclosure is shown in FIG. **14**. Here, vibration device **210** includes a first actuator **212** and a second actuator **214**, having respective moving masses **216** and **218**. FIG. **14** represents a two dimensional embodiment where two linear motion vibration actuators **212**, **214** are aligned with an xy plane. In this embodiment, it is not necessary for the actuators **212**, **214** to be orthogonal to each other. A1 and A2 are respectively the amplitudes of vibration of actuators **212** and **214**, while a1 and a2 are respectively the unit vectors specifying the direction of vibration of actuators **212** and **214**.

The unit vector a_1 is given by:

$$a_1 = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix} \tag{12}$$

where the angle α describes the orientation of actuator 1 relative to the x axis as shown in FIG. **14**. The unit vector a_2 is given by:

$$a_2 = \begin{bmatrix} \cos(\beta) \\ \sin(\beta) \end{bmatrix} \tag{13}$$

where the angle β describes the orientation of actuator 2 relative to the x axis as shown in FIG. **14**.

For a given vibration waveform the maximum magnitude of force vectors, $F_{1,max}$ and $F_{2,max}$, from actuators **212** and **214** in FIG. **14** can be given by equations:

$$F_{1,max}=A_1a_1 \tag{14}$$

$$F_{2,max}=A_2a_2 \tag{15}$$

When actuators **212** and **214** are vibrated synchronously and in-phase (e.g. with the same frequency and with zero phase difference), then the maximum force amplitude occurs at the same time. Thus the maximum combined force vector, $F_{combined,max}$, is given though superposition of the force vectors, and is given by:

$$F_{combined,max}=F_{1,max}F_{2,max} \tag{16}$$

A matrix of actuator directions, D_L, can be created where each of its columns is a unit vector that corresponds to the direction of vibration of a linear motion vibration actuator in a vibration device. For a vibration device with two linear motion vibration actuators, such as the one shown in FIG. **14**, the matrix D_L is given by:

$$D_L=[a_1|a_2] \tag{17}$$

where a_1 and a_2 are column vectors.

A matrix representation of the combined force is given by:

$$F_{combined,max} = D_L\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \tag{18}$$

where A_1 and A_2 are scalars. For the case of vibration in a plane, the vectors a_1 and a_2 will be 2×1 vectors and the matrix D_L will be 2×2.

When the direction matrix, D_L, is invertible then the amplitude of vibration in the individual actuators that corresponds to a desired combined force vector, $F_{combined}$, is given by:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = D^{-1}Fcombined \tag{19}$$

When the actuators are aligned orthogonally, then the direction matrix, D_L, is orthonormal and its inverse is given by its transpose as shown below:

$$D^{-1}=D^T \tag{20}$$

When the direction matrix, D_L, in not invertible because there are more vibration actuators than directions of force being controlled, then a pseudo inverse of matrix D_L can be used. For example, if there are 3 vibration actuators in the xy plane, and the control objective is only to control a two dimensional force, the D_L matrix is given by:

$$D_L=[a_1|a_2|a_3] \tag{21}$$

where a_1, a_2, and a_3 are 2×1 column vectors.

The pseudo inverse is described in "*Introduction to Linear Algebra*", 3rd Edition by Gilbert Strang, published in 2003 by Wellesley-Cambridge Press, the entire disclosure of which is incorporated by reference herein.

One method for calculating a pseudo inverse, D_L^+, is given by:

$$D_L^+=D_L^T(D_LD_L^T)^{-1} \tag{22}$$

In such a case the amplitude of vibration for each actuator can be given by:

$$\begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = D_L^+Fcombined \tag{23}$$

It is possible to specify the combined force vector, $F_{combined}$, in terms of a direction of vibration and amplitude. For a two dimensional embodiment the combined amplitude of vibration can be specified by the scalar $A_{combined}$, and the direction of vibration can be specified by an angle, theta, as shown in FIG. **14**. In this two dimensional embodiment $F_{combined}$ can be given by:

$$F_{combined} = A_{combined}\begin{bmatrix} \cos(\text{theta}) \\ \sin(\text{theta}) \end{bmatrix} \tag{24}$$

Thus, it can be seen that the amplitudes of vibration, A1 and A2, can be represented in terms of the direction of vibration, theta, combined amplitude of vibration, $A_{combined}$, and direction matrix, D_L, as given by:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = D_L^{-1}Acombined\begin{bmatrix} \cos(\text{theta}) \\ \sin(\text{theta}) \end{bmatrix} \tag{25}$$

Equation 25 provides the scalar magnitude of A_1 and A_2. When the sign of A_1 is different than the sign of A_2 then vibration waveform can be generated directly using the results of Eq. Avec. Alternatively, the waveform can be generated using absolute values of A_1 and A_2 but with one waveform completely out of phase with the other waveform. A sine wave is defined to be completely out of phase when it is 180 degrees out of phase. General waveforms are defined to be completely out of phase when the maximum positive amplitude of vibration of one waveform concedes with the maximum negative amplitude of the other waveform. A depiction of two actuators vibrating completely out of phase is shown in FIG. **13**. Two actuators vibrating completely out of phase are also considered to be in synchronized vibration.

It is also possible to specify the combined direction of vibration in terms of a unit vector, $a_{combined}$, as shown in FIG. **14**. The vector $F_{combined}$ can be given by:

$$F_{combined}=A_{combined}\times a_{combined} \tag{26}$$

Another configuration according to aspects of the present disclosure is a three dimensional configuration, where there are at least 3 linear motion vibration actuators as shown in FIG. **15**.

In the vibration device **220** of FIG. **15**, actuators **222**, **224** and **226** each include a moving mass **228**, **230** and **232**, respectively. The actuators **222**, **224** and **226** are preferably orthogonal to each other and aligned with an xyz coordinate system. In an alternative three dimensional embodiment the actuators are not necessarily orthogonal to each other; yet the force vectors of the actuators span the three dimensional vector space. With such an alternative, an arbitrary direction of three dimensional force can be generated. In the three dimensional cases, the combined direction of vibration can be specified by the 3×1 unit vector, $a_{combined}$. The three dimensional combined force can be given by the same equations for the 2 dimensional case, as shown below

$$F_{combined}=A_{combined}\times a_{combined} \tag{27}$$

where $a_{combined}$ and $F_{combined}$ are 3 dimensional vectors.

Vibration devices according to the present disclosure may include an arbitrary number of actuators in arbitrary locations and orientations.

FIG. **16** illustrates a vibration device **240** having a pair of actuators **242** and **244**. The actuators **242** and **244** include moving masses **246** and **248**, respectively. In this embodiment, vibration device housing **250** is configured as a hand held game controller for computer or video games. Linear motion vibration actuator **242** is shown as being located in the left handle and linear motion vibration actuator **244** is shown as being located in the right handle. The actuators **242** and **244** need not be orthogonal, and need not be in the same plane.

Another alternative embodiment of a vibration device according to the present disclosure is shown in FIG. **17**, where vibration device **260** includes a first linear motion vibration actuator **262** and a second linear motion vibration actuator **264**. As shown, the actuators **262**, **264** are located on top of each other. An advantage of such a configuration is that the actuators **262**, **264** create little torque about the center of the vibration device **260**, which may be desirable in some vibration applications.

In a variation of FIG. **17**, FIG. **18** illustrates a game controller **270** having two linear actuators, **272** and **274** disposed perpendicular to each other. The actuators **272** and **274** are preferably rigidly mounted to case **276** of a game controller. The actuators **272** and **274** could be mounted in a plane of any angle; however, they are preferably mounted in a horizontal plane of the case **276**. The actuators **272** and **274** do not have to be located one on top of the other; rather they can be attached to the same rigid body, such as the case **276** of a game controller. Of course, one could attach three or more linear actuators to the case **276**, preferably at right angles to each other to create force vectors than span the three dimensional space of the case **276**. Moreover, the actuators do not have to be at right angles to each other. Desirably, the actuators are positioned relative to one another with different orientations.

A further embodiment of a vibration device according to the present disclosure is shown in FIG. **19**. Here, vibration device **280** includes two linear motion vibration actuators, **282** and **284**, which are aligned in their orientation but separated by a distance D. Actuator **282** includes moving mass **286** and actuator **284** includes moving mass **288**. The actuators **282**, **284** may be vibrated such that the moving mass **286** in actuator **282** is at a negative extreme along the y axis when the moving mass **288** in actuator **284** has a positive extreme along the y axis. In this fashion the two actuators **282**, **284** generate a combined torque when vibrated in a synchronous fashion. The embodiment shown in FIG. **19** could be operated, in one example, such that the moving masses **286** and **288** move in the same direction when synchronized, and thereby generate a combined force along the y axis. In this fashion the configuration shown in FIG. **19** could be used to generate a combined torque, a combined force, or a combination of force and torque.

An alternative embodiment of a vibration device **290** in accordance with aspects of the present disclosure is shown in FIG. **20**. Here, three linear motion vibration actuators **292**, **294** and **296**, each having a moving mass, are orientated on an xy plane. In this embodiment it is possible to generate a combined force and a combined torque. It is also possible to independently control the combine force and torque by modulating the amplitude of vibration in each of the actuators **292**, **294** and **296**. The combined torque and force are superpositions of the forces and torques generated by each actuator. Since there are three actuators that can be controlled independently, the components of the force along the x axis, the force along the y axis, and the torque about a selected point on the xy plane can all be modulated independently.

In the vibration device embodiments described herein the vibration actuators may be attached to the vibration device in a rigid, a semi-rigid or a non-rigid fashion. Even when vibration actuators are attached in a non-rigid fashion to a vibration device, the vibration device is operable to transmit the superposition of forces from all vibration actuators. When vibration actuators are attached in a rigid fashion to a vibration device, the combined force applied by the vibration device becomes less dependent on the location where the vibration device transmits force and torques to other bodies. In addition, the more rigid the attachment between the vibration actuators and the vibration device, the more uniform the timing of the force superposition becomes at all points of the vibration device.

In an example, it is possible to attach the actuators directly onto a person's hand and body, for instance as shown in U.S. Pat. Nos. 6,275,213 and 6,424,333. In uses of the present disclosure where actuators are directly attached or indirectly coupled to the hand or body, the vibration force from each actuator may be felt directly at different locations on the body, yet a synchronized combined force vector can still be applied onto the body by synchronizing the operation of the actuators.

Vibration devices in accordance with the present disclosure can be built with rotary vibration actuators as well as with linear motion vibration actuators. In some cases the cost to manufacture rotary vibration actuators is less than the cost to manufacture linear motion vibration actuators. Thus, if cost is a factor, it may be desirable to utilize rotary vibration actuators in place of or in combination with linear motion vibration actuators. However, in order to generate synchronized vibration with rotary vibration actuators, it is necessary to control the rotary position of the actuators along with the rotary velocity.

A rotary vibration actuator may comprise, for example, a DC motor, a rotary solenoid, a rotary stepper motor, a servo motor, or other type of rotary actuator. One advantage of rotary actuators is their relatively low cost. The servo motor uses a position sensor and/or a velocity sensor for feedback. In some situations the rotary stepper motor may be more desirable because it allows for control of position and velocity without the use of a sensor.

FIG. **21** shows a rotary vibration actuator **300** suitable for use with the present disclosure. The actuator **300** includes an eccentric mass **302** coupled to a rotary actuator **304** along a shaft **306**. As the rotary actuator **304** is rotated, a centrifugal force is generated in the radial direction aligned with the eccentric mass **302** as shown by the vector CF in FIG. **21**.

Many existing vibrators utilize rotary vibration actuators with eccentric masses, but not with synchronized vibration. In accordance with the present disclosure, a pair of rotary vibration actuators can be configured to achieve a vibration force that is aligned with a single direction of motion. Accordingly, a pair of such rotary actuators can be used when a vibration force in a specified direction is required.

For instance, a vibration device according to the present disclosure can be built, by way of example only, with two rotary vibration actuators that rotate in opposite directions, as shown in FIG. **22**. As shown, the vibration device **310** includes a pair of rotary vibration actuators **312** and **314**, each having an eccentric mass **316** and **318**, respectively. Actuator **312** preferably rotates clockwise, and actuator **314** preferably rotates counterclockwise. In the orientation shown the centrifugal force vectors from both actuators are aligned with the y axis and superimpose to create a combined force vector, CVF, in the y direction.

With rotary vibration actuators it is possible to create synchronized vibration in an analogous fashion to the synchronized vibration described with linear motion vibration actuators. With rotary vibrating actuators, synchronized vibration is defined to occur where two rotary actuators rotate in approximately the same plane at the same angular velocity in opposite directions, and where the relative angle between the actuators is controlled, such that the actuator centrifugal force vectors align repeatedly in the direction of desired vibration force.

The direction of vibration force can be controlled with a pair of rotary (or rocking) vibration actuators by controlling the angle at which the centrifugal force vectors become aligned. Therefore, it is possible to control the direction of combined force with rotary actuators in a fashion analogous to how the direction of combined force can be controlled with multiple linear vibration actuators.

FIG. **23** shows the embodiment of two rotary vibration actuators as described with respect to FIG. **22**, wherein the actuators are controlled in synchronized vibration for a number of positions. As shown in FIG. **23**, the combined force vector, CFV, remains in the y axis, and its magnitude changes according to the rotary position of the actuators. The maximum combined force vector occurs when the centrifugal force from both rotary actuators are aligned.

An alternative type of rotary actuator suitable for use with the present disclosure is a rotary actuator with a pivoting mass. FIGS. **24**A-C illustrate respective front, side and bottom views of an exemplary pivoting actuator **400**, which includes a mass **402** operable to pivot relative to a rotary actuator **404**. The mass **402** is connected to the rotary actuator **404** via a shaft **406**. The center of mass of the mass **402** can be located anywhere on the body of the mass **402**. Thus, the center of mass may be concentric with the axis of rotation, or eccentric to the axis of rotation. The pivoting actuator **400** may be configured to function in a manner similar to the rotary vibration actuators discussed above.

As seen in FIGS. **25**A-C, the rotary actuator **404** may be affixed to a support **408**, which, in turn, may connect to another object (not shown). Preferably a spring device **410** couples the pivoting mass **402** to a support **412**, which may be the same or a different support than the support **408**. FIG. **25**A illustrates the pivoting actuator **400** when the spring device **410** is in a rest state when the pivoting mass **402** is in a central position.

The mass **402** may pivot in either a clockwise or counterclockwise manner. FIG. **25**B illustrates counterclockwise operation. Here, the spring device **410** is in a compressed state. In the present embodiment as shown, the spring device **410** is under a compression force that is primarily linear and is applied toward the right hand side of the figure. FIG. **25**C illustrates clockwise operation of the mass **402**. Here, the spring device **410** is in an uncompressed state in response to a force that is primarily linear and is applied toward the left hand side of the figure.

Vibration forces and/or torques can be generated with the pivoting actuator **400** as shown in FIGS. **25**A-C. The pivoting actuator **400** can be activated to pivot the pivoting mass **402** first clockwise and then counterclockwise, or vice versa. As the pivoting mass **402** rocks back and forth, the spring device **410** generates a vibration force, a torque, or both a vibration force and torque onto the object to which it is affixed via the support **408**. In this fashion, if the pivoting mass **402** has a center of mass concentric with the axis of rotation, the pivoting mass **402** can be used to generate a vibration torque. Also in this fashion, if the pivoting mass **402** has a center of mass eccentric with the axis of rotation, the pivoting mass **402** can be used to generate a vibration force.

Vibration forces and/or torques can be generated by moving a mass back and forth. It is possible to define the beginning of a vibration waveform as an instance at which a mass reverses its direction of motion. For linear actuators, the reversal of direction is a reversal of translation. For rotary actuators, the reversal of direction is a reversal of rotation. In general, the reversal of motion of a mass in an actuator may include both translation and rotation.

In actuators having a spring device attached to a moving mass, energy can be built up in the spring device, especially when the mass is moved back and forth close to a natural frequency of the mass and spring system. In such cases, the maximum vibration force can occur at the maximum deformation of the spring device, which can occur when the mass reaches its maximum excursion and reverses its direction. Accordingly, moving masses in two (or more) actuators that are operating in synchronized vibration, can reverse direction at approximately the same time.

An alternative method for generating vibration would be to operate the pivoting actuator **400** in a clockwise (or counterclockwise) direction and then deactivate the pivoting actuator **400** while allowing the spring device **410** to rotate the pivoting mass **402** in the counterclockwise (or clockwise) direction. This approach would allow one to use pivoting actuators and control circuitry that only operates in a single direction.

FIG. **26** illustrates a variation of the pivoting actuator **400**, namely pivoting actuator **400'**, which desirably includes the pivoting mass **402** operable to pivot relative to the rotary actuator **404**, and which is connected thereto via the shaft **406**. As above, the rotary actuator **404** may be affixed to the support **408**, which, in turn, may connect to another object (not shown). Preferably a first spring device **410***a* couples the pivoting mass **402** to a first support **412***a*, and a second spring device **410***b* also couples the pivoting mass **402** to a second support **412***b*. The supports **412***a* and **412***b* may be a single support, separate supports that are physically connected, or physically disconnected supports. One or both of the supports **412***a,b* may be the same or a different support than the support **408**.

One type of pivoting actuator **400** that could be employed is a DC motor. However, not all the components of the DC motor are necessary for this application, because the output shaft does not rotate continuously. Accordingly it is not necessary to have motor brushes, which can reduce cost as well as electrical power losses and frictional losses. In a preferred example, the pivoting actuator **400** may essentially include a stator and a rotor. The stator may be stationary and desirably contains permanent magnets and/or electromagnets. The rotor is operable to pivot and can contain permanent magnets and/or electromagnets. The polarity of the magnets in the stator and rotor can be configured so that activation of the electromagnets causes an electromagnetic torque to be exerted onto the rotating mass **402**.

In the embodiment of FIGS. **25**A-C, the spring device **410** is configured to operate in a generally linear fashion. However, In order to generate large magnitude of vibration forces with small actuators, it can be advantageous to utilize the resonance of a system. The embodiments shown in FIGS. **25**A-C have both a mass and a spring, and thus have a resonant frequency. If the actuator is excited at or close to this resonant frequency large amplitude vibrations can build up. However, it can be desirable to operate the vibration device at a range of frequencies. It is possible for a device to have a variable resonant frequency with use of nonlinear spring forces, as discussed in the aforementioned “Vibration Device” patent application. Accordingly, one could use a nonlinear spring in the vibration device to achieve larger amplitude of vibration over a range of frequencies.

It is possible to generate nonlinear spring force, even with use of a linear spring element. Consider the embodiment shown in FIG. **27**A. Here, pivoting actuator **420** has a mass **422** operable to pivot relative to a rotary actuator **424**. The mass **422** is connected to the rotary actuator **424** via a shaft **426**. The rotary actuator **424** may be affixed to a support **427**, which, in turn, may connect to another object (not shown). Preferably a spring device **428** couples the pivoting mass **422** to a support **427'**, which may be the same or a different support than the support **427**.

As shown in FIG. **27**A, the spring device **428** is desirably placed in-line with the pivoting mass axis. When the pivoting mass **422** is rotated a small amount about the center position very little lengthening occurs in the spring device **428**. Accordingly, the effective spring constant is low and the resonant frequency is low.

Low frequency operation is desirable in some situations, for instance in games that have low frequency effects. For instance, games may generate actions or events in the sub-200 Hertz range, such as between 15 and 150 Hertz. In certain cases the actions or events may be as low as 20-50 Hertz or lower, such as about 10-20 Hertz. Examples of such actions/events include gunshots, automobile related sounds such as a car spinning out of control, and helicopter related sounds such as the whirring of the rotor blades. Eccentric mass actuators may not be suitable to generate a haptic sensation in this frequency range, but pivoting actuators or linear actuators may generate such frequencies.

As the magnitude of rotation of the pivoting mass **422** increases, the lengthening of the spring device **428** increases as shown in FIGS. **27**B and **27**C. Accordingly, for larger amplitudes of rotation, the effective spring constant is higher and the natural frequency of the system is higher. In order to quickly ramp up the vibration amplitude when a nonlinear spring force is used, the excitation frequency can be varied so that it always matches the natural frequency of the vibration device.

FIG. **27**D illustrates a rotating actuator **430** having a rotating mass **432** coupled to rotary actuator **434** via shaft **436**. The rotary actuator **434** is desirably coupled to a support **437**, which, in turn, may connect to another object (not shown). In this alternative, a spring device such as a torsion spring **438** is attached between the rotating mass **432** and the rotary actuator **434**. As shown, one end or tang **439***a* of the torsion spring **438** is attached to the rotating mass **432**, and the other end or tang **439***b* is attached to the support **437** (or, alternatively, to the rotary actuator **434** itself). Torsion spring **438** may be employed because such spring devices permit a large degree of rotation of the rotating mass **432** relative to the rotary actuator **434** and the support **437**.

FIGS. **27**E and **27**F illustrate a further rotating actuator, namely rotating actuator **440**. The rotating actuator **440** includes a rotating mass **442** having a slot **443** therein, a rotary actuator **444**, and a shaft **446** coupling the rotating mass **442** to the rotary actuator **444**. The rotary actuator **444** is desirably coupled to a support **447**, which, in turn, may connect to another object (not shown). In this embodiment a pin **445** is held within the slot **443**. A spring device **448** is coupled at one end or tang **449***a* to the pin **445**. The spring device **448** is coupled at the other end or tang **449***b* to a support **447'**. The support **447'** is preferably different from the support **447**, or, alternatively, is preferably a different section of the support **447** from where the rotary actuator is coupled.

FIG. **27**E shows the spring device **448** in a "rest" position. FIG. **27**F shows the spring device **448** in a "compressed" position. Here, by way of example only, the rotating mass **442** may be rotating in a clockwise direction. As the rotating mass **442** rotates, the pin **445** moves relative to the slot **443**, but the spring device **448** remains in substantially the same orientation relative to the support **447'**. In this fashion, the force applied onto the fixed **447'** remains in relatively the same direction as the moving mass **442** rotates. It is possible to incorporate a gap between the slot **443** and the pin **445** that would allow for some rotation of the shaft **446** before the spring device **448** is extended or compressed from its rest position. The gap would create a non-linear force effect on the rotating mass **442**, which could aid in increasing the magnitude of vibration. The gap would allow the shaft **446** to more quickly reach higher speeds and for the rotating actuator **440** to more quickly build up rotating inertia.

While several types of actuators have been described above that may be used with the present disclosure, other types of actuators may also be employed so long as they can be controlled as described herein. For instance, piezoelectric devices without separate or distinct "moving" and "stationary" masses may be employed either alone or in combination with other actuator types to impart vibratory forces in the manners described herein.

FIG. **28** illustrates a synchronized vibration system **450**, which may comprise two vibration devices **452** and **454**, such as any of those of FIGS. **24**A-C, **25**A-C, **26** and/or **27**A-F. Of course, more that two vibration devices may be provided. The vibration devices **452** and **454** are preferably mounted onto a base plate **456** in a generally orthogonal manner as shown, although orthogonality is not required. The vibration device **452** is preferably a horizontal vibrator that desirably has a spring device **458** which applies primarily horizontal forces onto the base plate **456**. The vibration device **454** is preferably a vertical vibrator that desirably has a spring device **460** that applies primarily vertical forces onto the base plate **456**. As long as the directions of the vibration forces of the different vibration devices are not aligned, it is possible to control the combined direction of vibration using the synchronized vibration methods as described herein as well as in the aforementioned "Vibration Device" patent application.

An alternative embodiment of the present disclosure includes two rotary vibration actuators whose planes of vibration are not the same; however, in this case the two planes are not orthogonal to each other. In this embodiment, the component of centrifugal force from one actuator that can be projected onto the plane of the other actuator can be used to achieve a component of synchronous vibration.

In one example, two or more vibration devices may be mounted devices into a game controller, as shown in FIG. **29**A. Here, a game controller **470** includes a pair of vibration devices **472** and **474** mounted in both the right and left handles, respectively, of housing **476**. The directions of vibration of the vibration devices **472** and **474** are preferably not aligned, and thus it is possible to control the direction of vibration using the synchronized vibration approach discussed herein.

There are many orientations of both the rotary actuators and springs that can be used to achieve an embodiment where synchronized vibration is possible. For instance, the axis of rotation of both actuators can be aligned while the spring direction can vary, allowing an alternative configuration for synchronized vibration. FIG. **29**B illustrates a game controller **480** having a pair of vibration devices **482** and **484** within a housing **486** where the axes of the rotating shafts in both rotary actuators are aligned, yet the spring forces are not aligned.

FIG. **30** illustrates yet another variation similar to the rotary and pivoting vibration devices. Here, a rocking actuator **490** preferably includes a rocking weight **492** rotatable about a shaft **494**. Desirably, one end of the rocking weight **492** is operatively coupled via a first spring device **496***a* to a first support **498***a*. The same end of the rocking weight **492** is also desirably operatively coupled via a second spring device **496***b* to a second support **498***b*. The supports **498***a* and **498***b* may be a single support, separate supports that are physically connected, or physically disconnected supports. The rocking actuator **490** may be implemented in a device such as a game controller in any of the configuration described above.

A controller for synchronized vibration of a pair of rotary vibration actuators specifies the angular position of each rotating shaft, such that the angle where the centrifugal force vectors are aligned is the desired direction of force vibration and the angular position is incremented such that the rotational velocity matches the desired vibration frequency.

A system **500** having a controller for one or more vibration devices that use linear motion vibration actuators is shown in FIG. **31**. Vibration device controller **502** specifies the desired vibration effect and one or more driver circuit(s) **504***a*, **504***b*, . . . , **504**N provide the necessary power to actuators **506***a*, **506***b*, . . . , **506**N. While each actuator **506** is shown as being powered by a separate driver circuit **504**, it is possible for multiple actuators **506** to be driven by one driver circuit **504**.

The controller **502** may be, by way of example only, a microprocessor and the driver circuit(s) **504** may be, for instance, one or more electrical amplifiers. The controller **502** and drive circuit **504** may be integrated into a single microprocessor or single electrical circuit. The control method in this figure is for a configuration with N actuators, where N is an arbitrary number of actuators. Some of the figures showing various control methods in the instant application illustrate only two actuators. However, it should be understood that control methods according to the present disclosure can be extended to include an arbitrary number of actuators, as shown in FIG. **31**.

FIG. **32** shows a control method for two actuators. Here the controller **502** specifies the desired vibration amplitude, A, frequency, f, and phase, p, for each actuator **506**. The amplitude, frequency, and phase of actuator **506***a* (A1, f1, p1) may differ from the amplitude, frequency, and phase of actuator **506***b* (A2, f2, p2). The profile/waveform of the desired vibration force may be a sine wave, square wave, triangle wave, or other profile, such as is discussed above with regard to FIG. **1**. The actual vibration profiles/waveforms of the actuators **506***a,b* may differ from the desired vibration profiles due the dynamics of the drive circuits **504***a,b* and actuators **506***a,b*.

FIG. **33** shows a control method where the frequency of vibration, f, is the same for both actuators **506***a,b*. FIG. **34** shows a control method where the frequency of vibration, f, and the phase of vibration, p, are the same for both actuators **506***a,b*. In this embodiment, the actuators **506***a,b* are desirably driven synchronously such that the peak amplitude of vibration will occur approximately at the same time for both actuators **506***a,b*. The amplitude of vibration may differ between the actuators **506***a,b*.

FIG. **35** shows a control embodiment in accordance with the present disclosure where the vibration device controller **502** includes an internal direction and amplitude controller **508**, an internal frequency controller **510**, and an internal vibration controller **512**. The direction and amplitude controller **508** desirably specifies the combined vibration amplitude, Acombined, and the direction of vibration theta. The frequency controller **510** desirably specifies the vibration frequency, f. The vibration controller **512** uses the inputs of theta, Acombined, and f to output vibration commands to the individual actuators **506***a,b*. The vibration controller **512** is operable to output various waveforms including sine waves, square waves, triangle waves, or other profiles as discussed herein.

The output from the vibration device controller **502** shown in FIG. **35** provides the magnitude of vibration as a function of time to each drive circuit **504***a,b*. In the case where the profile of vibration is a sine wave, the amplitude of vibration for each actuator as a function of time is given by the equation shown below:

$$\begin{bmatrix} A_1(t) \\ A_2(t) \end{bmatrix} = D^{-1} Acombined \begin{bmatrix} \cos(\text{theta}) \\ \sin(\text{theta}) \end{bmatrix} \sin(\omega t + p) \tag{28}$$

Here, t is time and ω is the vibration frequency in radians per second. The parameter p is the phase of vibration and may be set to zero. The value of ω in terms of frequency f in vibrations per second is given by $\omega = 2\pi f$.

When the vibration actuators have a linear relationship between the command magnitude and the magnitude of vibration, the output $A_1(t)$ and $A_2(t)$ from equation 28 can be applied directly to the vibration actuators to generate a combined vibration direction corresponding to the angle theta. However some vibration actuators may have a nonlinear relationship between the command magnitude and the magnitude of vibration. For such nonlinear actuators it is possible to generate vibration in the direction theta by using a linearization function that adjusts the magnitude of A_1 and A_2 to compensate for the nonlinearity of the actuator, as shown in the following equation.

$$\begin{bmatrix} A_1(t) \\ A_2(t) \end{bmatrix} = \text{linearization\_function}\left\{ D^{-1} A_{combined} \begin{bmatrix} \cos(\text{theta}) \\ \sin(\text{theta}) \end{bmatrix} \sin(\omega t + p) \right\} \tag{29}$$

The linearization equation described above can be a lookup table or a scaling algorithm or other type of function.

The ability to control the direction of vibration over time, such as though use of equations 28 and 29, is an important advantage of the present disclosure. The ability to control vibration direction can be used in vibratory feeders to direct parts in a desired direction. In addition, there are numerous advantages of using the disclosure for haptic devices as described herein.

FIG. **36**A illustrates a system **550** showing the input of various input parameters of amplitude, phase and position (or time) for a pair of linear actuators. A computer **552** receives input of the parameters, which are preferably entered using a computer keyboard (not shown); however, the parameters also could be input using a graphical user interface, analog potentiometers, or many other means generally known to those skilled in the art. The appropriate output waveforms for linear actuators **554***a* and **554***b* are then computed using the computer **552**. Each waveform is preferably independent. While computation may be performed using an analog computer, a digital computer is preferred.

If a digital computer is used, the digital output for each actuator **554***a,b* is then preferably fed into respective digital-to-analog ("DAC") converters **556***a* and **556***b*, which convert the output to the appropriate analog waveform. The analog waveforms are then fed into the appropriate driver circuits **558***a* and **558***b*. Those skilled in the art could use other means to modulate the linear vibrations of each actuator **554***a* and **554***b*, for example via pulse width modulated ("PWM"). Varying the parameters produces an extremely broad range and rich set of haptic sensations for the end user.

In addition to creating varying force effects, one could control the direction of vibration—that is to say the direction of vibration could remain stationary. The resultant force effects can be of lower frequency than the frequency of vibration.

There are also useful applications for generating precise patterns of vibrations from simple parameters. Such patterns include circles, ellipses and straight lines. Furthermore, the amplitude and duration of the patterns may be precisely controlled over time. Moreover, a sequence of patterns may be generated as desired.

FIG. **36**B illustrates the system **550** where the input of various input parameters includes input of pattern number, amplitude, duration and start-time for the vibration device using compound vibrations. The parameters are preferably entered using a computer keyboard. The appropriate output waveforms for each linear actuator are then computed at computer **552**. As described above, the digital output for each actuator **554***a* and **554***b* is then fed into DACs **556***a* and **556***b* for conversion to the appropriate analog waveforms. The waveforms are then fed into the driver circuits **558***a* and **558***b*. Again, the various parameters produce an extremely broad and rich set of haptic sensations for the end user.

Each of the vibration devices described herein according to the present disclosure can be used as a haptic interface. Haptic interfaces provide force sensation to a user. Haptic interfaces include computer gaming controllers, robot controllers, surgical tool controllers, as well as other devices where a force sensation is provided to a user.

An embodiment **600** of the present disclosure with a haptic interface application is shown in FIG. **37**. In this embodiment a systems controller **602** provides force commands to a haptic interface **604** which generates forces which result in force sensations to user **606**. The systems controller **602** may be microprocessor, a central processing unit, an ASIC, a DSP, a game controller, an analog controller, or other type of controller or any combination thereof. The user **606** can input commands to the haptic interface **604** that are transmitted as user commands back to the system controller **602**. The user commands can be input through pressing buttons, moving joysticks, squeezing the haptic interface at various level forces, moving the haptic interface, applying force and torque onto the haptic interface and through other means.

In the embodiment shown in FIG. **37**, there is preferably a graphical display **608** which receives an image command from the system controller **602** and displays a visual image to the user **606**. The graphical display **608** may be, for instance, a computer monitor, a television monitor, an LCD display, a plasma display, a combination of light sources, or other type of means for generating a graphical image. A haptic interface application can also be implemented without a graphical display **608**.

A haptic interface application can include a simulation of a virtual environment or representation of a real environment to the user **606**. A systems controller method of control can be based upon this real or virtual environment. Typical simulated environments include games, driving and flight simulations, surgical simulations, and other types of simulations. Typical real world environments include control of robots and remote machines, long distance interactions, and other types of environments. It is often desirable that a haptic interface provide force sensations that correlate with the real or simulated environment in which the haptic interface is being used.

Another embodiment **620** having a haptic interface application is shown in FIG. **38**. This embodiment is similar to the one of FIG. **37**, and includes a systems controller **622**, which provides force commands to a haptic interface **624** that generates forces which result in force sensations being received by user **626**. A graphical display **628** is also provided for receiving image commands from the system controller **622** and for displaying a visual image to the user **626**.

In the embodiment of FIG. **38**, the haptic interface **624** desirably includes a vibration device **630** having vibration actuators (not shown), a vibration controller **632**, driver circuits **634** which drive the vibration device actuators, and an input device **636**, which can detect user input and which can include buttons, joysticks, and pressure sensors. The components of the haptic interface **624** may be of any of the configurations described herein. In this embodiment the graphical display **628** preferably presents a two dimensional image. The graphical display **628** shows an object of interest at a direction specified by the angle theta. It is may be desirable that the force sensation felt by the user **626** correspond to the image on the graphical display in terms of direction, such as theta, and other attributes.

The embodiment shown in FIG. **38** can be utilized so that the force sensations felt by the user **626** are generated by the vibration device controller **632** specifically to correspond to the image on the graphical display **628**. The vibration device controller **632** may specify one or more of the amplitude of vibration, Acombined, direction of force, theta, and frequency of vibration, f, as described above. The values of Acombined, theta, and/or f can be selected to correspond to the image on the graphical display **628** and the environment being used by the system controller **622**. The complete force effect (including frequency, amplitude, combined direction of force and torque, and duration of force effect) generated by the vibration device may correlate events within a graphical computer simulation. Several examples of such operation follow.

A first example involves the simulation of a user firing a gun. In this simulation, the vibration device controller **632** could specify the angle theta to represent the direction of a gun firing, the amplitude of vibration, Acombined, to represent the amplitude of the gun recoil, and the frequency of vibration, f, to represent the frequency of bullets leaving the gun.

A second example involves an impact between objects. In this simulation the vibration device controller **632** may specify the angle theta to represent the direction of impact, and the amplitude of vibration, Acombined, to represent the amplitude of impact.

A third example involves driving a vehicle. In this simulation the vibration device controller **632** could specify the angle theta to represent the direction of vehicle motion, the frequency of vibration, f, to represent the frequency of vehicle vibration as it drives over bumps in the road or the speed of the vehicle, and the amplitude of vibration, Acombined, to represent the amplitude of bumps in the road.

A fourth example involves a car or spacecraft spinning out of control. In this simulation the vibration device controller **632** could specify an angle theta that represents the vehicle's orientation. To represent the vehicle spinning, the angle theta can vary over time. The rate at which the angle theta can be different than the vibration frequency. Typically the frequency at which a vehicle spins would be significantly lower than typical vibration frequencies.

An algorithm that can be used to create the vehicle spinning described above varies the direction of vibration continually. The direction of vibration may be rotated at a rate of β radians per second, using the equation below:

$$\begin{bmatrix} A_1(t) \\ A_2(t) \end{bmatrix} = D^{-1} A_{combined} \begin{bmatrix} \cos(\beta t) \\ \sin(\beta t) \end{bmatrix} \sin(\omega t + p) \tag{30}$$

Equation 30 illustrates that the frequency of direction change, β, can be modified independently from the frequency of vibration ω. A user such as user **606** or **626** can sense both the frequency of vibration and the direction of vibration. In this fashion, sensations at both the β and ω frequencies can felt by the user. It is possible to set the frequency β much lower than the frequency ω, thereby overcoming a limitation of known devices. By way of example only, ω may vary between 10 Hz and 100 Hz while β may be on the order of 1 Hz. In another instance, β may vary from between about 5% to 20% of ω. Of course, in other instances ω and β may be similar or the same, or, alternatively, β may be larger than ω. All of these examples will depend on the specific effect that is desired.

Low frequency operation is desirable in some situations, for instance in games that have low frequency effects. For instance, games may generate actions or events in the sub-200 Hertz range, such as between 1 and 150 Hertz. In certain cases the actions or events may be as low as 2 Hertz or lower, such as about 0.5-1 Hertz. Examples of such actions/events include gunshots, automobile related sounds such as corresponding to a car spinning out of control, and helicopter related sounds such as the whirring of the rotor blades. A traditional eccentric mass actuator may not be suitable to generate a haptic sensation in this frequency range; however, two or more vibration actuators operated in synchronized vibration may generate such frequencies.

β is not limited to any particular rate or range of rates. For instance, β may be a relatively low rate to represent a slow spinning action, e.g., of a car spin out at less than 10 miles per hour, or β may be a relatively high rate to represent a fast spinning action, e.g., of a car spin out at a speed in excess of 40 miles per hour. Similarly, ω is not limited to any particular frequency of vibration. Preferably, ω is set within a range of frequencies that can be felt or otherwise detected by a user.

Equation 30 may be modified by changing the vibration profile from a sine wave to a square wave, triangle wave, or other profile. In addition, the amplitude of vibration, Acombined, can be varied over time. The frequencies β and ω can also be varied over time. In this fashion a wide range of force effects can be created.

Vibration actuators can be used to provide haptic sensations either through synchronized vibration or otherwise. Actuators can be vibrated without synchronization when there is no need to convey directional information, and then the actuators can be switched to synchronous vibration when there is a need to convey directional information though the haptic interface.

Many linear motion vibration actuators take advantage of resonance to achieve relatively high level of forces with low power requirements. However, to achieve these high levels of forces a number of vibration cycles have to occur before the peak magnitude of vibration occurs. In addition when the actuator is shut off, the moving mass in the actuator may continue to oscillate for a number of cycles. Thus the dynamics of the actuator prevents instantaneous response of the actuator to increase or decrease the magnitude of vibration.

When synchronous vibration is used to control the direction of combined force, the actuator dynamics may limit the speed at which the direction of combined force can be changed. One of the examples presented above describes implementation of a haptic force sensation that corresponds to the spinning of a car. However, the actuator dynamics may limit the rate at which such spinning effect can be generated. As will be described in detail below, it is possible to provide a method that can increase the rate at which the direction of force can be changed for a system of vibration actuators that are synchronously vibrated.

Equation 25 above defines the required amplitude of vibration of actuators to achieve a combined force direction corresponding to an angle theta. For a given actuator in a vibration device, the required amplitude of vibration is defined as Ades, which indicates the desired amplitude of vibration of that actuator. If the actuator is at rest or at a lower level of vibration than Ades, then it may be desirable to initially drive the actuator at a higher level of vibration to more quickly raise the amplitude of vibration to Ades. Conversely if the actuator is already vibrating at an amplitude higher than Ades it may be desirable to initially drive the actuator at a lower level or even brake the actuator to more quickly lower the amplitude of vibration to Ades. These variations in the amplitude at which the actuator is driven are defined as corrections to the commanded vibration magnitude.

One method of determining the proper corrections to the vibration magnitude is to model the dynamics of the actuator. This approach allows one to predict the dynamic states of the actuator and optimal commands to most quickly generate the desired amplitude of vibration.

An alternate method of determining the corrections to the vibration magnitude does not require a dynamic model of the actuator or explicitly predicting the dynamic states of the actuator. In this method a counter is maintained to track the recent number of vibrations of the actuator and the corresponding commands sent to the actuator during these recent vibrations. The command to the actuator at the k^{th} vibration is given by the following equation:

$$A_{com\_k} = A_{des\_k} + A_{cor\_k}$$

A_{des_k} represents the desired actuator amplitude for the k^{th} vibration of the actuator. A_{cor_k} represents the correction to the command for the k^{th} vibration. And A_{com_k} represents the actual amplitude of the command sent to the actuator for the k^{th} vibration.

If the desired amplitude at the k^{th} vibration is greater than the amplitude during the previous vibration, then most likely the vibration level needs to be increased. Accordingly, the correction to the command at vibration k, A_{cor_k}, can be chosen to be proportional to the difference between the current desired amplitude, A_{des_k}, and the previous commanded amplitude A_{com_k-1}. An equation which described this approach for calculation A_{cor_k} is:

$$A_{cor\_k} = K^*(A_{des\_k} - A_{com\_k-1}) \tag{31}$$

Here, K is a gain chosen based upon actuator performance. This same equation works for reducing the magnitude of vibration quickly. When A_{des_k} is less than the value of A_{com_k-1} it indicates that most likely the level of vibration needs to be reduced and the correction A_{cor_k} is negative. If the large reduction in vibration amplitude is commanded, then the negative magnitude of A_{cor_k} may be greater than A_{des_k} and the actual command sent to the actuator, A_{com_k}, will be negative resulting in braking of the moving mass in the actuator.

Another approach to correcting the magnitude of vibration takes into consideration the two previous commanded amplitudes, and is given by the following equation:

$$A_{cor\_k}=K_1*(A_{des\_k}-A_{com\_k-1})+K_2*(A_{des\_k}-A\mathrm{com}_{k-2}) \quad (32)$$

Here K_1 is a gain that corresponds to the k−1 vibration command, and K_2 is a gain that corresponds to the k−2 vibration command. In a similar fashion even more prior commands can be incorporated into the correction algorithm. The following equation shows how "m" prior commands can be incorporated into an actuator command.

$$A_{cor\_k}=K_1*(A_{des\_k}-A_{com\_k-1})+K_2*(A_{des\_k}-A_{com\_k-2})+ \dots +K_m*(A_{des\_k}-A_{com\_k-n}) \quad (33)$$

Alternative methods of control for multiple vibrating actuators may include modified synchronization. One method of modified synchronization is for one actuator to vibrate at a frequency that is an integer multiple of the vibration frequency of another actuator. FIG. **39** is a plot **650** presenting two vibration profiles, **652** and **654**, showing such a control method. The vibration frequency of profile **654** is twice the vibration frequency of profile **652**. The beginning of cycles of vibration can be controlled to occur at the same time only ever other cycle for profile **2**. Thus the superposition of peak amplitudes only occurs ever other cycle for profile **654**. This modified synchronization method can be applied for arbitrary integer multiples of vibration frequency, arbitrary vibration profiles, and an arbitrary number of actuators.

One advantage of such a modified synchronization method is that multiple vibration frequencies can occur at the same time while still providing for some superposition or peak amplitudes. The superposition of peak amplitudes allows for control of direction of vibration, in a similar fashion to how the direction for vibration is controlled for synchronized vibration. With this modified method of synchronized vibration, it is possible to specify the direction of combined force only during a portion of the vibration cycle. Nevertheless, a direction component to the vibration can be controlled in the duration close to the time where the superposition of peaks occurs. Close to the time at which there is superposition of peaks in the vibrations, the combined force vector, $F_{combined}$, can be approximated by:

$$F_{combined}=a_1A_1+a_2A_2 \quad (34)$$

Here, a_1 and a_2 are the unit vectors aligned with the directions of actuator 1 and actuator 2, respectively. A_1 and A_2 are the amplitudes of force of actuator 1 and actuator 2, respectively, near the duration of the superposition of peaks. By modifying the amplitudes A_1 and A_2 it is possible to modify the amplitude and direction of the combined force vector, $F_{combined}$. A similar approach can be used when there are more than two vibration actuators.

If there are two or more vibrating actuators where repeatedly the peak amplitude of force of these vibrating actuators occurs at approximately the same time, then the combined direction of force of these actuators can be controlled near the time when these repeated peak amplitudes occur. In this case, the combined direction of force can be controlled by modifying the amplitude of vibration of the actuators.

An alternative modified synchronization is to drive two vibration actuators at the same frequency but one vibration actuator at a phase where its peak magnitude of force occurs when a second vibration actuator is at zero force, which is at 90 degrees out of phase for a sinusoidal vibration. In such a modified synchronization the combined force direction rotates in a circle or ellipsoid during each vibration period.

Additional methods for modified synchronization of vibration may include the superposition of profiles as described in the "Jules Lissajous and His Figures" ("Lissajous"), appearing in chapter 12 of "Trigonometric Delights" by Eli Maor, published in 1998 by Princeton University Press. The entire disclosure of Lissajous is hereby incorporated by reference. Lissajous describes how profiles can be combined through various combinations of frequencies, phases, amplitudes, and profiles to generate a wide range of output figures. These are also known as Bowditch curves. Lissajous also describes how geometric shapes can be created from multiple vibration sources. These combinations of vibrations can be applied to haptic devices and vibration devices in accordance with aspects of the present disclosure. Thus, the concepts of superposition described in Lissajous can be applied by vibration actuators to yield a wide range of force sensations.

Electric actuators often require a driver circuit separate from a controller. The driver circuit provides sufficient current and voltage to drive the Actuators with the necessary electrical power. A wide range of driver circuits have been developed for electrical actuators and specifically for vibration actuators, and are known to those skilled in the field. Such driver circuits include linear drivers, PWM drivers, unipolar drivers, and bipolar drivers. A circuit block diagram for a vibration actuator **700** according to the present disclosure includes a vibration controller **702**, a driver circuit **704**, and an actuator **706**, as shown in FIG. **40**.

The vibration controller **702** shown in FIG. **40** can be located on the vibration device itself or could be located remotely, where the vibration signals are transmitted to the driver circuit **704** through wired or wireless communication.

It is often desirable to control a vibration device or actuators from a digital controller such as a microprocessor or other digital circuit. Digital control circuits often have low level power output, and therefore require a higher power driver circuit to drive an actuator. In addition, low cost digital controllers often have digital outputs, but do not have analog outputs. To simplify the vibration controller circuitry and lower cost, the vibration signal can be a binary logic directional signal which signals the moving mass to move either forward or backwards. In this configuration, the vibration signal can be in the form of a square wave to generate the desired vibration effect. Even with such a square wave control signal, the actual motion and vibration force of the vibration actuator will most likely not follow a square wave exactly due to the dynamics of the actuator.

To further simplify the vibration controller circuitry and lower cost, the amplitude of the vibration signal can be modulated with a PWM signal, where the duty cycle of the signal is proportional to the amplitude of vibration. An embodiment **710** with such a digital vibration controller **712** for one actuator **716** is shown in FIG. **41**. In this embodiment, the output of the digital vibration controller **712** includes an amplitude signal in PWM form and a direction signal, for instance in the form of a logic bit, both of which preferably are sent to a driver circuit **714**. The driver circuit **714**, in turn, sends electrical power to the actuator **716**.

Digital control circuitry can be used to control a complete vibration device in synchronized vibration. In synchronized vibration the frequency and phase of two or more actuators are the same. Accordingly, a single square wave can be used to control the direction of the vibration actuators that are in synchronized vibration. The amplitude of vibration can be controlled independently for each actuator, with separate PWM signals.

FIG. **42** shows an embodiment **720** where a vibration device controller **722** generates one directional signal ("dir"), which may be in the form of a square wave. The dir signal is preferably provided to a pair of drive circuits **724***a* and **724***b*. The vibration device controller **722** desirably generates separate amplitude signals, A1 and A2, in PWM form to the drive circuits **724***a,b* for a pair of actuators **726***a* and **726***b*. The vibration device controller **722** preferably includes a direction and amplitude controller **728**, a frequency controller **730** and a vibration controller **732** as in the embodiment described above with regard to FIG. **35**. The direction and amplitude controller **728**, the frequency controller **730** and the vibration controller **732** may be configured in hardware, software, firmware or a combination thereof, and may be implemented either as separate components or processes, or may be implemented as a single component or process.

The embodiment **720** of FIG. **42** may be used to control in synchronous vibration the vibration devices with two actuators, for instance as described above with regard to FIGS. **10-20**. Embodiment **720** can also be used to vibrate two or more actuators completely out of phase, which occurs during synchronized vibration when equation 25 provides results with the sign of A1 being different than the sign of A2. To vibrate two actuators completely out of phase, the binary direction signal dir can be inverted for one of the actuators. The inversion of the directional signal dir can occur at a driver circuit **724***a* or **724***b*, or the vibration controller **732** can output two directional signals, with one being the inverse of the other. The case where two actuators are being driven completely out of phase is shown in FIG. **13**.

Electric actuators in accordance with the present disclosure can be driven with unipolar or bipolar drivers. A unipolar driver will generate current in an actuator in a single direction. A unipolar driver is well suited for actuators where the moving mass is ferromagnetic and an electromagnetic coil only generates attractive magnetic forces, such as the actuator **150** shown in FIG. **9**. One example of a unipolar driver circuit is a Darlington array, such as the ULN2803A DARLINGTON TRANSISTOR ARRAY manufactured by Texas Instruments.

A bipolar driver can generate current in two directions. Bipolar drivers are well suited for actuators where the moving mass is magnetic and where reversing the direction of current in an electromagnetic coil can reverse the direction of force on the moving mass. Examples of such actuators are presented in FIGS. **5**A-B through **8**A-B. One example for a bipolar driver circuit is an H bridge, such as the L298 manufactured by ST Microelectronics. Alternative H bridges are the 3958 and 3959 drivers manufactured by Allegro Microsystems.

In vibrating circuits it can be advantageous to increase power output of the driver circuits through use of a charge pump capacitor as used in 3958 and 3959 drivers manufactured by Allegro Microsystems. It can also be advantageous to incorporate a capacitor in series with a linear motion vibrating actuator to benefit from a resonance effect and temporary storage of energy in the capacitor, as described in the aforementioned U.S. Patent Application entitled "Vibration Device."

As detailed herein, vibration actuators can be used in a variety of methods to create haptic effects. Vibration actuators can be operated continuously throughout the duration of a specified haptic effect, or can be pulsed on and off during the haptic effect. By pulsing vibration actuators on and off the user feels only a small number of vibrations, then feels a pause, and then the vibration resumes. In this fashion it is possible to generate secondary sensations associated with the frequency of pulsing the actuators on and off. Examples of how such pulse effects can be used are described in U.S. Pat. Nos. 6,275,213 and 6,424,333.

Any of the actuators described herein may be used in accordance with the present disclosure to produce a wide variety of haptic effects. While some actuators such as linear actuators and rocking mass actuators may be particularly suited for low frequency operation, all actuators herein may provide synchronized feedback. Such feedback may be employed in games, virtual reality equipment, real-world equipment such as surgical tools and construction equipment, as well as portable electronic devices such as cellular phones and pagers. By way of example only, cellular phones and pagers may implement different vibration effects to identify different callers or different actions. Synchronized vibration may provide directional feedback, for instance, with the impact or recoil of a gun in a game, or to distinguish between frontal and side impacts in driving games. Synchronized vibration may also provide a continual rotation of a vibration force vector in a game to simulate a car spinning out of control. Synchronized vibration may also be used in endless other applications and situations to provide a rich haptic experience to a user.

As mentioned above, other aspects of the disclosure include General Synchronized Vibration. General Synchronized Vibration differs from non-synchronized vibration in that the frequency and phase of multiple vibration forces are controlled. Embodiments with multiple Vibration Actuators that are not controlled with the General Synchronized Vibration approach will often have inconsistent frequency, amplitude, or relative phase between the actuators. With General Synchronized Vibration the frequency and phase of the Vibration Actuators may vary during the start-up and transitions between various waveforms. However, once the actuators are synchronized, each actuator is controlled to a specific frequency and phase.

Often each actuator is controlled to a fixed frequency and phase for a given duration of time. This duration of time depends on the application, but is typically longer than the period of the highest frequency vibration force that is being synchronized. In haptic applications this duration of time is typically along enough for a person to sense the effect. However, there are some implementations of General Synchronized Vibration where the desired waveform of vibration varies quickly, such as a quickly changing direction used to provide a sensation of spinning. In such quickly varying waveforms, the desired frequency and phase of a vibration actuator may be changing in a duration that is shorter than the period of the vibration of that actuator. A common characteristic of General Synchronized Vibration is that the frequency and relative phase of multiple vibration actuators are explicitly controlled to desired values rather than randomly selected values.

In General Synchronized Vibration there is typically a consistent correlation between frequency and phase of the actuators and desired vibration effects. For example, a haptics effect library for software developers may have a routine labeled "spin," which generates a sequence of desired frequency and phase for a plurality of Vibration Actuators. Each time the spin effect is executed, a similar sequence of frequency and phase and generated by the plurality of Vibration Actuators.

Embodiments of this disclosure include a Vibration Device comprised of multiple Vibration Actuators mounted onto a mounting platform such as a base plate, sub-frame, housing, or enclosure. For example the mounting platform could be the housing of a game controller, or the housing of a Vibration Actuator. The mounting platform transfers force and torque between the Vibration Actuators and thereby allows the vibration forces and torques to be superimposed upon each other. The mounting platform is preferably rigid, but can also be relatively rigid component, or a semi-rigid component. The mounting platform could be made of separate pieces. The mounting platform could include components of an object upon which vibration forces are being applied. For example if multiple Vibration Actuators are mounted onto a person's arm or other body parts and forces are transmitted from these actuators through the arm or body parts, then the arm or body parts can serve as the mounting platform. This disclosure pertains to any configuration where the forces and torques from multiple Vibration Actuators can be vectorially combined to generate a net vibration force, vibration torque, or vibration force and torque.

The mounting platform is typically attached to a number of items such as battery, control circuit board, and the stationary parts of the Vibration Actuators including housing and stator. The combined mass of the mounting platform and items that are attached to it is defined as a "Reference Mass". The vibration force and torques are transferred from Vibration Actuators to the Reference Mass. If the mounting platform is able to move, the vibration forces may shake the Reference Mass. Typically the Reference Mass is in contact with an "External Object", and forces and torques are transmitted between the Reference Mass and the External Object. For example, a game controller held in a user's hand would transfer forces and torques from the game controller's Reference Mass onto a user's hands, which in this case is an External Object. The mounting platform may be attached to the Earth, which would also be an External Object. A Vibration Device attached to the Earth is sometimes termed a "Shaker" or a "Shaker Device".

A preferred embodiment uses two aligned LRAs, as shown in FIG. **43**. LRA **1102***a* and LRA **1102***b* are attached to mounting platform **1100** and are aligned in the axis of vibration that they generate. Each LRA has a moving mass, **1108**, and a housing **1106** which is attached to the Mounting platform **1100**. This configuration of vibration actuators is referred to as an LRA Pair. The vibration forces from each LRA are combined together through the mounting platform **1100**. The vibration force generated by LRA **102***a* is designated as F1 and the vibration force generated by LRA **1102***b* is designated by F2.

For the embodiment shown in FIG. **43**, one method of generating an asymmetric vibration force is to operate LRA **1102***b* at twice the frequency of LRA **1102***a*, with a specified phase difference of either 90 or −90 degrees. The vibration forces in such an embodiment with sinusoidal vibrations can be given by:

$$F_1=B_1 \sin(\omega_1 t+\phi_1)$$

$$F_2=B_2 \sin(\omega_2 t+\phi_2)$$

Where $\omega_2=2\omega_1$

$\phi_1=0$, and $\phi_2=-90$

The combined force for the LRA Pair is given by:

$$F_{LRA\_Pair}=B_1 \sin(\omega_1 t+\phi_1)+B_2 \sin(\omega_2 t+\phi_2) \tag{35}$$

Typically it is not critical to control vibration effects relative to absolute time. Accordingly, when implementing the vibration effect described in Eq. 35 above, it is not critical to control both the phase ϕ_1 and ϕ_2, but rather the relative phase between the two actuators. Therefore in some implementations one could set phase ϕ_1 to zero and control only ϕ_2. Alternatively one could directly control the phase difference between the actuators. In this application typically the phases of all the actuators are shown in the equations. However, without loss of utility only the relative phase of the actuators can be controlled. Thus the phase of Vibration Actuators 2, 3, 4, etc. would be controlled relative to the phase of actuator 1; thereby eliminating the need to control the phase of actuator 1 relative to absolute time.

A feature of this disclosure includes the use of superposition of synchronized vibration waveforms. When multiple vibration forces are generated on a single vibration device, the Combined Vibration Force for the device is the superposition of the multiple waveforms. An example with two synchronized sine waves described by Eq. 35 is shown in FIG. **44**. As shown, waveform 2 has twice the frequency of waveform 1. The phase of both waveforms is set such that at a time of zero the peaks of both waveforms have their maximum value in a positive direction, and the forces magnitudes are added together (also referred to as constructive interference or positive interference).

Furthermore, at the time when waveform 1 is at its negative peak then waveform 2 is at a positive peak, and the forces magnitudes are subtracted from each other (also referred to as destructive interference or negative interference). Due to this synchronization the combined vibration waveform is asymmetric, meaning that the force profile for positive force values is different than the force profile for negative force values. In the asymmetric waveform shown in FIG. **44** there is a higher peak positive force and a lower peak negative force. In haptic applications the larger force in the positive direction can generate more of a force sensation than the lower magnitude force in the negative direction, even though the duration of force in the negative direction is longer. In this fashion asymmetric vibrations can be used to generate a haptic cue in a specific direction with a vibration device.

In an LRA, a moving mass moves relative to the actuator housing, and a restoring spring transfers force between the moving mass and the actuator housing. The force imparted by an LRA onto a mounting platform is a combination of the force from the restoring spring, and the electromagnetic force between the stator and moving mass. The restoring spring can be, for example: a mechanical spring or a magnetic spring. As resonance builds up in an LRA, the magnitude of the spring restoring force increases and becomes the dominant portion of the actuator force. Accordingly, the peak force imparted by a LRA onto the mounting platform typically occurs at or near the peak excursion point of the moving mass.

In FIG. **43** the moving masses are graphically depicted as towards the right side of the LRAs to indicate actuator forces being applied to the right. Accordingly, when the embodiment shown in FIG. **43** is controlled to follow the waveform described by Eq. 35, then the moving mass of LRA **1102***a* is at its peak excursion to the right at the same time when the moving mass of LRA **1102***b* is at its peak excursion to the right resulting in a large combined force to the right, yet when the moving mass of LRA **1102***a* is at or near its peak excursion to the left then the moving mass of LRA **1102***b* is at or near its peak excursion to the right (since it is vibrating at twice the frequency) resulting in force cancellation and a low combined force to the left.

Thus, in this embodiment the timing of the moving masses is an indication of an asymmetric vibration waveform. In FIG. **45**, the embodiment shown in FIG. **43** is shown at various time steps as it implements the vibration waveform shown in FIG. **44**. In **45**, the top LRA vibrates at twice the frequency and generates lower forces that the bottom LRA, the position of the moving masses indicates the forces generated by each LRA, and the combined force vector is shown between the LRAs. Each time step in FIG. **45** is labeled according to the period, T, of the slower LRA.

In the embodiment shown in FIG. **43**, the alignment of the actuators does not have to be precise. Indeed, in haptic applications having the two actuators are not precisely aligned may not deter from the primary haptic effect that is being generated.

A variation of this embodiment is shown in FIG. **46**. The actuators **1102***a* and **1102***b* are attached directly to each other to provide an even more compact configuration. Also the LRAs can share housings, shafts, power supplies, and other components to make the device even more compact.

Another variation of this embodiment is shown in FIG. **47**. The actuators **1102***a* and **1102***b* are attached in line with each other. In this embodiment, the forces of each LRA are collinear, and create no net torque along the axis of the LRAs. This embodiment is useful where pure force output is desired without any torque output.

The timing of vibration force within a Vibration Actuator can be correlated with a number of physical properties. For example, in many LRAs a spring applies a restoring force onto a moving mass and the vibration force is largely correlated with the position of the moving mass. In ERMs the direction of the vibration force largely correlates to the angular position of a rotating eccentric mass. Linkage mechanisms can be used to generate vibrations, such as a slider-crank vibration actuator **1110** shown in FIG. **48**, where a rotating motor **1114** moves a mass **1120** back and forth. With such linkages the vibration force can be correlated with the acceleration of a moving mass. Since the vibration force can be correlated with a number of physical properties, General Synchronized Vibration can also be characterized by control of the frequency and phase of the position or acceleration of moving masses within Vibration Actuators.

A feature of this disclosure includes combining vibration waveforms from multiple Vibration Actuators to generate a more complex vibration waveform. The asymmetric vibration described by Eq. 35 and shown in FIG. **44** is only one such type of combined vibration waveform. A more general embodiment shown FIG. **49** has a set of N LRAs all aligned with the same axis. According to one aspect, in General Synchronized Vibration a plurality of Vibration Actuators are synchronized in phase and frequency, and in some cases amplitude. A wide range of vibration effects can be generated by controlling the frequencies and phases of all N actuators.

A vibration force, F, is in a repeated cycle over a period T when F(t+T)=F(t). The vibration force of an ith actuator in a repeated cycle can be given by:

$$F_1(t+\Delta_i+T_i)=F_i(\Delta_i+t),$$

where Δ_i is the phase and T_i is the period of the ith actuator. For the embodiment shown in FIG. **49**, there is a set of N LRAs all aligned with the same axis. If all actuators are operated at set frequencies and phases, then the combined vibration force can be given by:

$$F_{AlignedSet}=F_1(\Delta_1+t)+F_2(\Delta_2+t)+\ldots+F_N(\Delta_N+t) \tag{36}$$

In the general case, the waveform shapes of F_i can be a wide range of waveforms including sine waves, triangle waves, square waves, or other waveforms. In some embodiments, the frequency of the actuator with the lowest frequency is defined as the fundamental frequency, ω_1, and the remaining actuators vibrate at integer multiples of the fundamental frequency. In these embodiments the period of the fundamental frequency is given by T_1 and the remaining vibration periods are given by such that:

$$T_1=2T_2, T_1=3T_3, \ldots T_1=NT_N$$

When all the vibration actuators vibrate at integer multiples of the fundamental frequency, then the combined waveform has a repeated waveform with a period of the fundamental frequency. The fundamental frequency is also referred to as the first harmonic.

One method of implementing General Synchronized Vibration is to use sinusoidal vibrations in each actuator of an aligned set, and use Fourier Waveform Synthesis to select the phase, frequency, and amplitude of each actuator to approximate a desired vibration waveform. For a set of N aligned actuators with sinusoidal waveforms, the combined force of an Aligned Set, $F_{AlignedSetFourier}$, is given by:

$$F_{AlignedSetFourier}=B_1\sin(\omega_1 t+\phi_1)+B_2\sin(\omega_2 t+\phi_2)+\ldots+B_N\sin(\omega_N t+\phi_N) \tag{37}$$

A wide range of additional waveforms can be synthesized from a set (a plurality) of vibration waveforms. Fourier synthesis is a method whereby an arbitrary waveform can be approximated from a combination of sine waves, including both symmetric and asymmetric waveforms. It is advantageous to use actuators vibrating at frequencies that are integer multiples of the frequency of vibration of other actuators. The lowest frequency in the set is referred to as the fundamental frequency or the first harmonic, the second harmonic is twice the fundamental frequency, the third harmonic is three times the fundamental frequency, and so on.

An advantage of using harmonics is that all the waveforms in the set repeat at the period of the fundamental frequency, thereby providing a repeating waveform profile of the combined waveform. In many vibration applications each vibration actuator generates a force with a repeated waveform that has a zero DC component and the combined force is described by Eq. 37. Accordingly, the combined vibration force does not have a DC component. Fourier synthesis is widely used in create a wide range of waveforms. One example waveform is a Sawtooth waveform, which creates a sudden change of force in one direction. In this manner, the Sawtooth waveform can be used to generate directional haptic cues. When the set of waveforms consists of three sine waves, the Sawtooth waveform can be generated with the first harmonic at relative amplitude 1, the second harmonic is at relative amplitude of ½, and the third linear sine wave with a relative amplitude of ⅓. With Fourier waveform synthesis, arbitrary waveforms can be approximated including both symmetric and asymmetric waveforms. When using Fourier waveform synthesis, both constructive and destructive interference can occur for both the positive and negative forces amplitudes.

An operating advantage of an LRA is to use resonance to generate high magnitude vibration forces from a relatively low power input, and an LRA can be designed and manufactured to have a specific resonant frequency by optimizing its spring stiffness and moving mass. In embodiments of General Synchronized Vibration, it can be advantageous to select a set of LRAs with resonant frequencies that correspond to at least some of the harmonics of a desired waveform. For example for a vibration device such as that in FIG. **49** with a set of n LRAs, the first LRA **1102**a could have a specified resonant frequency of ω_1, the second LRA **1102**b could have a specified resonant frequency of $2\omega_1$, the third LRA could have a specified resonant frequency of $3\omega_1$, and so on through the nth LRA **1102**n.

Although LRAs are generally designed to operate at their resonant frequency, one can operate LRAs at other frequencies with lower amplitude force output per input command signal. Since lower amplitude force output is typically required at higher harmonics, once could build a Vibration Device with LRAs that all have the same resonant frequency, but operate them at different frequencies. For example for a vibration device with a set of 2 LRAs, both LRAs could have a specified resonant frequency of $(3/2)\omega_1$, where the first LRA is driven at ω_1, and the second LRA is driven at $2\omega_1$. In this configuration both LRAs are amplifying the input signal, but less than if they were driven at the resonant frequency of the LRAs, which is $(3/2)\omega_1$ for this example.

Asymmetric Vibration waveforms are useful for generating directional haptic cues, and can be synthesized using Fourier synthesis. For instance, an example of a method for selecting frequency, phase, and amplitude of sinusoidal vibrations to generate a high level of asymmetry is discussed below. Vibration parameters are specified for a set of 2, 3, and 4 actuators. In addition a process is presented for identifying parameters for waveforms with a high level of vibration asymmetry for any number of actuators. It should be noted that high levels of asymmetry may be achieved even if the values specified by this example are only approximately implemented. For instance, in the case of superposition of two sine waves, if there is a 30% error in the amplitude of vibration then 90% of desired asymmetry effect will still be realized.

Fourier synthesis allows one to approximate an arbitrary waveform with a superposition of sinusoidal waves. However, it is advantageous in some applications to generate asymmetric waveforms that have higher peak magnitudes in the positive direction than in the negative direction (or vice versa). The question then becomes what is the best function to approximate that will maximize the amount of asymmetry for a given number of superimposed sine waves? It is of special interest to consider asymmetric waveforms that have a zero DC component and thus can be composed solely of sine waves. Waveforms with a zero DC component can be used to generate vibrations from a set of vibrators since each vibrator will typically have a zero DC component. An asymmetric pulse train is illustrated in FIG. **50**. The pulse-train is just one example of an asymmetric waveform, but it is a useful example. For the pulse-train to have a zero DC component, the area above the axis. Thus:

$$W \cdot P = (T - W)V, \text{ and}$$

$$V = \frac{W \cdot P}{(T - W)},$$

where W is the pulse width, V is valley amplitude, T is period of repeated pulse, and P is peak amplitude.

The amount of asymmetry in a pulse-train can be defined by the percentage increase of P over V. One could increase the amount of asymmetry by reducing W, which would generate a thin and high pulse. However, if W is too small, the waveform would not be well-approximated with a small number of sine waves. Accordingly, an analytical question is, "What is the optimal value of W for a waveform composed of N sine waves?"

FIG. **51** illustrates a pulse-train with zero DC component. Given this waveform, one may find its Fourier coefficients according to the Fourier series:

$$f(t) = a_0 + \sum_{n=1}^{N} (a_n \sin(2\pi nt) + b_n \cos(2\pi nt)),$$

where f(t) is an arbitrary waveform and when $a_0=0$ it have zero DC component. The Fourier coefficients can be calculated by multiplying both sides of the above equation by $\sin(2\pi nt)$ or $\cos(2\pi nt)$ and then canceling out terms. The coefficients are:

$$\frac{a_n}{2} = \int_0^T f(t)\sin(2\pi nt)\,dt$$

$$\frac{b_n}{2} = \int_0^T f(t)\cos(2\pi nt)\,dt$$

$$a_0 = \int_0^T f(t)\,dt = 0$$

The equation for a_0 holds if the DC component is zero. For the pulse waveform above, a_n is given by:

$$\frac{a_n}{2} = \int_0^W P\sin(2\pi nt)\,dt + \int_W^T (-V)\sin(2\pi nt)\,dt$$

$$= \frac{-P\cos(2\pi nt)}{2\pi n}\Bigg]_0^W + \frac{V\cos(2\pi nt)}{2\pi n}\Bigg]_W^T$$

$$\frac{a_n}{2} = \frac{-P\cos(2\pi nW)}{2\pi n} + \frac{P}{2\pi n} + \frac{V\cos(2\pi nT)}{2\pi n} - \frac{V\cos(2\pi nW)}{2\pi n}$$

$$\frac{a_n}{2} = \left(\frac{1}{2\pi n}\right)(P + V\cos(2\pi nT) - (P + V)\cos(2\pi nW))$$

In a similar fashion:

$$\frac{b_n}{2} = \int_0^W P\cos(2\pi nt)\,dt + \int_W^T (-V)\cos(2\pi nt)\,dt$$

$$\frac{b_n}{2} = \left(\frac{1}{2\pi n}\right)((P + V)\sin(2\pi nW) - V\sin(2\pi nT))$$

By substituting in V from the equation above, the result is:

$$\frac{a_n}{2} = \frac{-PT\cos(2\pi nW) + PW\cos(2\pi nT) + PT - PW}{2\pi nT - 2\pi nW}$$

$$\frac{b_n}{2} = -\frac{PW\sin(2\pi nT) - PT\sin(2\pi nW)}{2\pi nT - 2\pi nW}$$

FIG. **52** is a flow diagram illustrating a process for maximizing asymmetry. As shown in the flow diagram, the process includes selecting a number of sine waves, and then guessing (estimating) values for W. Fourier coefficients are then calculated, and the time domain of the wave, f(t), is generated according to the equation set forth above. The amount of asymmetry in f(t) is then calculated. The process may be repeated with different values for W, and the value for W is selected that gives the most asymmetry.

Fourier coefficients can be represented by a_n and b_n as:

$$f(t) = a_0 + \sum_{n=1}^{N} (a_n \sin(2\pi nt) + b_n \cos(2\pi nt))$$

An alternative representation using sine waves and phase is:

$$f(t) = A_0 + \sum_{n=1}^{N} A_n \sin(2\pi nt + \phi_n)$$

To relate the two representations, the addition of sines formula:

$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$$

may be used with:

$$\alpha=2\pi nt$$

$$\beta=\phi_n$$

$$A_n \sin(2\pi nt+\phi_n)=A_n \cos(\phi_n)\cdot\sin(2\pi nt)+A_n \sin(\phi_n)\cos(2\pi nt)$$

Let

$$a_n = A_n\cos(\phi_n) \text{ and } b_n = A_n\sin(\phi_n)$$

$$\therefore A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{A_n^2\cos^2(\phi_n) + A_n^2\sin^2(\phi_n)} = A_n$$

where

$$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

In one scenario, the process shown in FIG. **52** was implemented for a range of sine waves according to the table below.

TABLE I

NACT	W	Asym	A_1	ϕ_1	A_2	ϕ_2	A_3	ϕ_3	A_4	ϕ_4
2	0.33	100%	1	30°	0.5	−30°				
3	0.25	189%	1	45°	0.71	0°	0.33	−45°		
4	0.2	269%	1	54°	0.81	18°	0.54	−18°	0.25	−54°

The variable "NACT" in Table I is used to define the number of sine waves since it can also represent the number of actuators. For two sine waves, an asymmetry of 100% can be achieved, which indicates there is twice the magnitude in the positive direction (or vice versa). Higher numbers of sine waves can provide even higher amounts of asymmetry as shown in Table I. One example is shown in FIG. **44**. Other examples are shown in FIGS. **53-55**.

General Synchronized Vibration can be performed with a set of non-sinusoidal waveforms. Even without use of Fourier synthesis, asymmetric waveforms can be generated by synchronizing the waveforms to create positive interference of two or more waveforms in one direction, and negative interference of two or more waveforms in the opposite direction. Embodiments with non-sinusoidal waveforms can still have the peaks of two or more waveforms occur simultaneously with positive interference in one direction and also occur simultaneously with negative interference in the opposite direction.

FIG. **56** shows two triangular waveforms that are synchronized together. Profile **1112***a* has twice the amplitude of profile **1112***b*, while profile **1112***b* vibrates at twice the frequency of profile **1112***a*. The peaks of profile **1112***a* and **1112***b* occur simultaneously, at times with positive interference and at times with negative interference. The combined waveform of profile **1112***a* and **1112***b* will generate an asymmetric waveform in a similar fashion that the combined waveform in FIG. **44**.

To create an especially distinct vibration effect, some LRA vibration actuators can be operated at an amplitude high enough to push the moving mass into the travel stops, thereby creating an impact force during each oscillation. The impact with the travel stops will generate a vibration waveform that is not sinusoidal. Multiple such actuators can be synchronized together to generate positive and negative interference as instances of impacts of masses with travel stops. This configuration can generate sharp peaks of vibration force, where direction of vibration is controllable. These sharp peaks of vibrations could be used to generate haptic sensations corresponding to impacts such as simulating the recoil of a gun. A wide range of vibration effects can be generated with non-sinusoidal vibrations. Examples are presented herein that use sine wave vibration waveforms, with the understanding that similar approaches could be generated with other waveforms.

One waveform that can be simulated is referred to as a "missing fundamental" waveform, which takes advantage of a phenomenon of human perception. As explained in "Music and Connectionism" by Peter M. Todd, D. Gareth Loy, MIT Press 2003, humans may perceive that a sound contains pitch of a certain frequency even though that frequency is not present in the sound if the sound contains higher frequencies that are integer multiples of the low frequency. In haptic applications, low frequency vibrations may be difficult to generate due to size and power constraints, while it may be easier to generate higher frequency vibrations. A vibration waveform can be generated that does not contain a desired low frequency, but does include higher frequencies at integer multiples of the desired low frequency. A person may perceive the desired low frequency vibration, just as they perceive the missing fundamental in a sound. The perception of a missing fundamental in vibration can be enhanced by including audio or visual effects at the desired low frequency.

The embodiment shown in FIG. **57** can generate asymmetric torques about the mounting platform. A pair of LRAs **1116***a* and **1116***b* are mounted towards the top of the mounting platform **1100**. A second pair of LRAs **1118***a* and **1118***b* are mounted towards the bottom of mounting platform **1100**. When the top pair of LRAs is operated with the same magnitude but opposite direction force than the bottom pair, a pure torque is generated on the mounting platform. When both the top and bottom pair vibrate with an asymmetric waveform, such as that shown in FIG. **44**, then the torque vibration is also asymmetric and can apply a higher peak torque in the clockwise direction than the counter-clockwise direction (or vice versa). Furthermore, the amplitude of the asymmetric torque vibration may be controlled by proportionally controlling the peak force in each LRA.

LRAs generate vibration forces along an axis and thus are described as "Linear Force Actuators." Other Linear Force Actuators include slider-crank vibrators, rack and pinion vibrators, linear actuators that do not use resonance, pistons, and solenoids. Rocking actuators and pivoting actuators (such as described in U.S. patent application Ser. No. 11/476,436) generate forces that are approximately along an axis and for many applications can be considered Linear Force Actuators. Indeed, any embodiment described herein as employing LRAs can also be implemented with Linear Force Actuators or other actuators that generate forces that are approximately along an axis.

A controller for General Synchronized Vibration of a pair of Linear Force Actuators is shown in FIG. **58**, which could control embodiments such as that shown in FIG. **43**. A vibration device controller generates commands of frequency, f, commanded amplitudes, Ac, and commanded phase pc. A driver circuit generates the voltage and current that drives the actuators. The driver circuit may output a waveform of a sine wave, square wave, triangle wave, or other waveform. The actuator may generate a force waveform that is similar to the waveform output of the driver circuit. Alternatively, the actuator may generate a force waveform that differs from the waveform output of the driver circuit. For example, the driver circuit may output a square wave but the actuator may generate a force that is mostly a sine wave due to the physics of the actuator.

Both LRA and ERM Vibration Actuators take some time to ramp up to speed to generate their maximum force output. Embodiments described herein include controllers that may or may not synchronize the actuators during the ramp up period. In addition, a Vibration Device may be commanded to transition from one vibration effect to another vibration effect. During this transition time interval, the controller may or may not synchronize the actuators.

A vibration device controller can be a microprocessor or other programmable device. For each actuator in the vibration device, the vibration device controller can modify the frequency of vibration, the phase of vibration, the amplitude of vibration, or any combination of these parameters. The ability to change these parameters allows for a single vibration device to generate a wide range of waveforms.

The phase and amplitude of the force output of a Vibration Actuator depends on both the control signal and the physical characteristics of the actuator. For example there is often a phase lag between the control signal and the force output of the actuator. To distinguish between the waveform of the actuator outputs and the waveform of the control signal, the subscript "c" notation is used to designate the control waveform. Thus the commanded amplitudes, Ac, and the commanded phase pc are not necessarily a direct correlation to the actual amplitude and phase of the actuator force. For example, the command voltage, V, of a vibration device controller of an LRA actuator driven with a sinusoidal voltage signal at a frequency ω, with a command phase of ϕc, and a voltage peak magnitude A_c, given by:

$$V=A_c \sin(\omega t+\phi_c) \tag{38}$$

However, due to the phase lag inherent in the actuator and frequency response of the actuator, the steady state force output of the actuator, F_a, may be given by:

$$F_a=A \sin(\omega t+\phi) \tag{39}$$

The phase lag is the difference between ϕ and ϕ_c. The frequency response is reflected in the ratio between A_c and A. Both the phase lag and the frequency response are functions of the actuator physics that can vary with vibration frequency, and which is often represented by an actuator specific Bode plot. For effective implementation of synchronized vibration it can be advantageous to take into consideration the phase lag inherent in each vibration actuator. This can be done by adding an equal but opposite phase offset to the controller waveform so that the actuator phase lag does not impact synchronization.

One method to implement this offset is to use a look up table, Bode plot, or algorithm for each actuator that determines the appropriate phase offset for a given vibration frequency. In addition, it can be advantageous to use a lookup table, Bode plot, or algorithm to determine the required voltage magnitude needed to generated the desired vibration force magnitude. The Fourier synthesis approach and the approach of matching positive and negative peaks of vibration described herein are implemented in reference to the actual phase of the actuator force output rather than the phase of the waveform from the actuator drive circuits. In order to simplify notation herein, the phase lag due to the actuator physics is generally not included in the equations relating to synchronization. Rather a more compact notation is used which represents the vibration force output, F, with the understanding that the appropriate command signal is generated to provided that output. The command signal includes the necessary phase lag and magnitude adjustment as needed based upon the actuator physics. The magnitude control can be implemented with a voltage, current, PWM signal of voltage or current, or other type of command used to drive said actuator. The Fourier synthesis approach and the approach of matching positive and negative peaks of vibration describe specific target frequency and phase of vibration for actuators within the vibration device; however, even if these target frequency and phase are not exactly met, the overall vibration effect often is close enough to the desired waveform to achieve a desired effect.

Due to manufacturing variations, two actuators that are built on the same assembly line may have different physical characteristics that affect their Bode plot, including phase lag, amplitude characteristics or resonant frequency. In some embodiments a sensor or sensors can be used to detect the phase of an actuator, the amplitude of vibration of an actuator, or the amplitude and phase. Such a sensor could be an optical sensor, Hall-effect sensor or other type of sensor that detects when a moving mass passes the midpoint or other point of vibration. One such embodiment is shown in FIG. **59**, where a sensor **1128** is integrated into to a Linear Force Actuator **1124** and detects when the moving mass **1126** reaches passes a midpoint position. A sensor integrated into an actuator can provide continuous, continual or periodic measurement of actuator performance and be used to update calibration parameters while the device is in use and does not require a specified calibration period.

Another method of sensing is to attach actuators **1124***a* and **1124***b* to the Mounting Platform **1100** of the vibration device **1134** as shown in FIG. **60**. This sensor **1136** could be an accelerometer or other sensor that measures the combined motion or combined force of the mounting platform.

The sensor measurements can be used to self-calibrate the vibration devices. A test pattern can operate each actuator separately to identify the actuator phase lag, force amplitude characteristics, and resonant frequency. These characteristics can be used to update a lookup table, Bode plot, or algorithm used to generate the voltage commands to the actuators. The combined force of multiple actuators can also be measured to confirm that the desired force effects are being achieved. Accordingly, the vibration device controller can use the sensor measurements to update the commanded amplitude, phase, and frequency as shown in FIG. **61**.

Embodiments of the disclosure also include configurations with multiple sets of aligned vibration actuators. One such configuration is shown in FIG. **62** that includes two sets of actuators. Set 1 consists of two LRAs **1138***a* and **1138***b* that are both aligned with the x axis of the vibration device **1134**. Set 2 consists of two LRAs **1140***a* and **1140***b* that are both aligned with the y axis of the vibration device. Set 1 generates force F_{S1B1} from LRA **1138***a*, and generates force F_{S1B2} from LRA **1138***b*. Set 2 generates force F_{S2B1} from LRA **1140***a*, and generates force F_{S2B2} from LRA **1140***b*.

In the embodiment shown in FIG. **62**, the combined vibration force is the vector sum of all the vibration actuators. Using the notation of U.S. patent application Ser. No. 11/476,436, a_1 and a_2 are unit vectors aligned with the forces from set 1 and set 2 respectively. In one control approach for the embodiment shown in FIG. **62**, the waveforms of both sets are controlled to have similar shapes but with different magnitudes. Magnitude coefficients are designated by the variable A, where the scalar A_1 multiplies the waveform of set 1 and the scalar A_2 multiplies the waveform of set 2. The combined force vector, $F_{combined}$, with this control approach with sinusoidal waveforms is given by:

$$F_{combined}=a_1A_1(B_1\sin(\omega_1t+\phi_1)+B_2\sin(\omega_2t+\phi_2))+a_2A_2(B_1\sin(\omega_1t+\phi_1)+B_2\sin(\omega_2t+\phi_2)) \quad (40)$$

As described in U.S. patent application Ser. No. 11/476,436, there are methods for selecting the magnitude of A_1 and A_2 that will generate a desired direction for the vector $F_{combined}$, yet these methods may only specify the axis of vibration and not whether the magnitude of force is positive or negative and thus limit the range of unique direction of vibrations to a range of 180 degrees. According to one aspect of the disclosure, an embodiment allowing control of the direction of vibration in all 360 degrees of the plane of the Mounting Platform, may have the following parameter relationships:

$\omega_2=2\omega_1$;

$\phi_1=0$ and $\phi_2=-90$ for a direction between −90 and +90 degrees;

$\phi_1=0$ and $\phi_2=90$ for a direction between 90 and 270 degrees;

A_1 and A_2 specified by equation 19 above

Numerous other embodiments are possible with multiple sets of aligned vibration actuators. Each set of aligned actuators can generate an arbitrary waveform, $p_{AlignedSet}$. Embodiments of synchronized vibrations created from arbitrary shaped profiles are described above. Many such embodiments show a single actuator generating each waveform. However, it is also possible to have a set of aligned actuators create these waveforms. Therefore, such embodiments can be expanded to include configurations where a set of aligned actuators take the place of a single actuator. In these configurations, the arbitrary waveform profiles would take the form of the arbitrary waveform, $p_{AlignedSet}$, as discussed herein.

Accordingly, embodiments of asymmetric vibration include 3D configurations and non-orthogonal configurations. An example of two non-orthogonal LRA Pairs is shown in FIG. **63**. These LRAs can generate waveforms in desired directions throughout the xy plane. The actuators in each aligned set can be LRAs, rocker actuators, and other sets of actuators that generate approximately linear forces.

An equation describing the combined force vector for M aligned sets with all sets having similar shaped waveforms but potentially different magnitudes is given by:

$$F_{combined}=a_1A_1(p_{AlignedSet})+a_2A_2(p_{AlignedSet})+\ldots+a_MA_M(p_{AlignedSet}) \quad (41)$$

The approaches used to determine the values of A described above can be applied to these configurations as well. A variety of Lissajous vibration patterns are also described above, including lines, circles, ellipses, parabolas, etc. Asymmetric vibration waveforms can be used to produce larger peak forces during one part of the Lissajous vibration pattern than another part.

Turning to another aspect of the disclosure, an ERM is depicted in FIG. **64**. A basic ERM includes a motor **1204**, a shaft **1208**, and an eccentric mass **1206**. The motor **1204** could be a DC brushed motor, a DC brushless motor, an AC induction motor, stepper motor, or any other device that turns electrical energy into rotary motion. The shaft **1208** is a power transmission element that transmits the rotary motion of the motor into rotary motion of the eccentric mass. However, alternate power transmission methods could be any means of transmitting the rotary motion of the motor **1204** into rotary motion of the eccentric mass **1206**, such as a belt, gear train, chain, or rotary joint. The eccentric mass **1206** could be any body that spins on an axis that is not coincident with its center of mass. Furthermore, the power transmission element may include geometry such that the axis of rotation of the eccentric mass **1206** is not necessarily coincident or parallel to the rotation axis of the motor **1204**, and the eccentric mass **1206** does not necessarily rotate at the same angular velocity as the motor **1204**.

One method of generating vibration forces is with an ERM where an eccentric mass is attached to motor shaft. As the motor rotates, centrifugal forces are generated onto the motor. General Synchronized Vibration can be applied to multiple ERMs by controlling the frequency and phase of rotation of the eccentric masses. FIG. **65** shows one embodiment for a vibration device **1200** that uses an arbitrary number M ERMS; the first two being ERM **1210***a*, **1210***b* and the last being **1210***m*. All ERMs are attached to a mounting platform **1202** and the combined vibration force of the device is the vector sum from all ERMs.

For the ith ERM, Ai is the amplitude of the vibration force, ωi is the frequency of vibration, and ϕi is the phase of vibration. The combined vibration force of the ERMs in FIG. **65** is given in the x and y coordinates by:

$$F_{Ex}=A_1\cos(\omega_1t+\phi_1)+A_2\cos(\omega 2t+\phi_2)+\ldots+A_M\cos(\omega_Mt+\phi_M)$$

$$F_{Ey}=A_1\sin(\omega_1t+\phi_2)+A_2\sin(\omega_2t+\phi_2)+\ldots+A_M\sin(\omega_Mt+\phi_M)$$

FIG. **66** shows one embodiment for a vibration device **1200** that uses four ERMS **1212***a*, **1212***b*, **1214***a*, and **1214***b*. All four ERMs are attached to a mounting platform **1202** and the combined vibration force of the device is the vector sum from all four ERMs.

The force and torque imparted by an ERM onto a mounting platform are due to a combination of the centrifugal force from the rotating eccentric mass, the torque between the stator and rotor of the motor and other inertial forces such as gyroscopic effects. As the speed of the ERM increases the centrifugal force increases and typically becomes the dominant portion of the vibration force. Accordingly, once an ERM has sped up, the vibration force imparted by an ERM onto the mounting platform is close to the centrifugal force imparted by the rotating eccentric mass.

In one embodiment, the ERMs are configured in counter-rotating pairs, where each ERM in a pair has the same eccentric mass and operates at the same angular speed but the ERMs rotate in opposite directions from each other. FIG. **4** ERM shows such an embodiment with a first counter-rotating pair consisting of ERM **1212***a* and ERM **1212***b*. The combined vibration force of just this first pair is given by:

$$F_{E1x}=A_1\cos(\omega_1 t+\phi_1+\phi_1+\sigma_1)+A_1\cos(-\omega_1 t-\phi_1+\sigma_1)$$

$$F_{E1y}=A_1\sin(\omega_1 t+\phi^1+\sigma_1)+A_1\sin(-\omega_1 t-\phi_1+\sigma_1)$$

The phase difference between ERM **1212***a* and ERM **1212***b* is represented by two variables, ϕ_1 and σ_1, where ϕ_1 represents a temporal phase and is half of the difference in overall phase and σ_1 represents a geometric angle and is half of the average of the overall phase difference. For an ERM the magnitude of the vibration force, A, is equal to $mr\omega^2$, where m is the mass, r is the radius of eccentricity, and ω is the speed of angular rotation in radians per second. Through trigonometric identities, this combined vibration force vector of the first ERM pair can be represented by the equation below. In this configuration, the force from a single counter-rotating pair generates a sinusoidal vibration force aligned with an axis of force direction defined by the angle σ_1.

$$F_{E1}=2A_1\cos(\omega_1 t+\phi_1)\begin{bmatrix}\cos(\sigma_1)\\ \sin(\sigma_1)\end{bmatrix} \tag{42}$$

The embodiment in FIG. **66** has a second counter-rotating pair formed by ERM **1214***a* and ERM **1214***b*, with both ERMS having the same eccentric mass as each other and operating at the same angular speed as each other but in opposite directions. This second counter-rotating pair generates a combined vibration force of:

$$F_{E2x}=A_2\cos(\omega_n t+\phi_2+\sigma_2)+A_2\cos(-\omega_2 t-\phi_2+\sigma_2)$$

$$F_{E2y}=A_2\sin(\omega_2 t+\phi_2+\sigma_2)+A_2\sin(-\omega_2 t-\phi_2+\sigma_2)$$

In one control method, σ_1 and σ_2 are set equal to the same value, σ, and therefore both ERM pairs generate a vibration along the same axis and the combined vibration force vibration force vector of all four ERMs is given by:

$$F_E=2A_1\cos(\omega_1 t+\phi_1)\begin{bmatrix}\cos(\sigma)\\ \sin(\sigma)\end{bmatrix}+2A_2\cos(\omega_2 t+\phi_2)\begin{bmatrix}\cos(\sigma)\\ \sin(\sigma)\end{bmatrix} \tag{43}$$

In another control method, σ_2 is set equal to $\pi+\sigma_1$ and therefore both ERM pairs generate a vibration along the same axis but the contribution from the second ERM pair has a negative sign. With this method the combined vibration force vibration force vector of all four ERMs is given by:

$$F_E=2A_1\cos(\omega_1 t+\phi_1)\begin{bmatrix}\cos(\sigma)\\ \sin(\sigma)\end{bmatrix}-2A_2\cos(\omega_2 t+\phi_2)\begin{bmatrix}\cos(\sigma)\\ \sin(\sigma)\end{bmatrix} \tag{44}$$

There are similarities between the application of General Synchronized Vibration to Linear Force Actuators and ERMs. In both cases, the combined vibration force can be composed of a superposition of sine waves, and in both cases it is possible to implement asymmetric vibrations. One embodiment asymmetric vibration uses the relative magnitudes and phases for superposition of two sinusoidal waves. In this embodiment, the amplitude of the fundamental frequency is twice that of the second harmonic. For the embodiment shown in FIG. **66**, a configuration for high asymmetry is shown in Table II below. The Geometric Angle, σ, can be selected arbitrarily based upon the desired direction of vibration. The eccentricity of the second ERM pair is represented relative to the eccentricity of the first ERM pair. The speed of rotation of the second ERM pair is twice the speed of rotation of the first ERM pair. It should be noted that high levels of asymmetry may be achieved even if the values specified in Table II are only approximately implemented. For example, in the case of superposition of two sine waves, if there is a 30% error in the amplitude of vibration, then 90% of desired asymmetry effect may still be realized.

TABLE II

ERM	Centrifugal Force Magnitude	Eccentricity	Geometric Angle, σ (radians)	Frequency of Rotation (radians/sec)	Temporal Phase, ϕ (radians)
1212a	A_1	m_1r_1	σ	ω_1	0
1212b	A_1	m_1r_1	σ	$-\omega_1$	0
1214a	$(1/2)A_1$	$(1/8)m_1r_1$	σ	$2\omega_1$	0 (or π)
1214b	$(1/2)A_1$	$(1/8)m_1r_1$	σ	$-2\omega_1$	0 (or π)

Steps of General Synchronized Vibration are shown in FIG. **67** for the case of a configuration shown in Table II. The time, t, is represented in terms of the period of the fundamental frequency, where $T_1=2\pi/\omega_1$. As seen in the uppermost illustration of FIG. **67**, at time t=0, the forces of all ERMs are aligned with the axis of vibration in the positive direction, and the position of the eccentric masses are all aligned in the same orientation. Accordingly, at t=0 the combined vibration force has a large magnitude. At $t=T_1/4$ as shown in the center illustration, the combined force vector is in the negative direction along the axis of vibration, yet the negative magnitude is not at a peak value since contribution only occurs from ERM **1214***a* and ERM **1214***b*, while the forces from ERM **1212***a* and ERM **1212***b* cancel each other out. At $t=T_1/2$, as shown in the bottom illustration, the combined force vector is also in the negative direction along the axis of vibration, yet the negative magnitude is not at a peak value since there is negative interference between the first ERM pair (ERM **1212***a* and **1212***b*) and the second ERM pair (ERM **1214***a* and **1214***b*). At $t=T_1/2$ the forces of the first ERM pair are in the opposite direction of the forces from the second ERM pair, and the orientation of the eccentric masses of the first ERM pair is 180 degrees opposite the orientation of the eccentric masses of the second ERM pair. Accordingly, asymmetric vibration is generated with a larger peak force occurring along the positive direction aligned with the axis of vibration. As shown in Table II the temporal phase of ERMs **1214***a* and **1214***b* can also be set to π, in which case asymmetric vibration will occur with a larger peak force along the negative direction aligned with the axis of vibration.

Embodiments are possible with a plurality of ERM pairs, as shown in FIG. **68** which has N ERM pairs; the first two pairs being **1216***a* and **1216***b*, and the last pair **1216***n*. In one control method the first ERM pair **1216***a* is rotated at a fundamental frequency, the second ERM pair **1216***b* is rotated at twice the fundamental frequency, and so on through all N pairs with the Nth pair **1216***n* rotating at N times the fundamental frequency. Using Fourier synthesis it is possible to approximate a wide range of waveforms.

In the embodiment shown in FIG. **68**, each ERM within a pair can have the same eccentricity, and each pair can be controlled so that one ERM in the pair rotates in the opposite direction of the other ERM with the same rotational speed. Asymmetric vibrations can be generated that have a higher peak force in a direction relative to the peak force in the opposite direction. High amounts of asymmetry can be generated using the process discussed above with regard to FIG. **52** (and Table I), which specifies magnitudes and phases for each harmonic sine wave. The magnitude of vibration of an ERM is the product of the eccentricity, mr, and the angular velocity, ω, squared. Accordingly, the eccentricity of the ith ERM as a function of the relative sine wave amplitude is given by:

$$m_n r_n = (A_n/A_1) m_1 r_1 / n^2 \tag{45}$$

The phases may be represented relative to the starting time of a specific waveform of pulse-trains being approximated. In some implementations it is more convenient to set the phase of the first harmonic to zero and represent the phases of the other harmonics relative to the first harmonic. An equation that converts the phase of the nth harmonic, ϕ_n, to a phase of the nth harmonic relative to the first harmonic, is given by:

$$\phi_{rn} = \phi_n - (\omega_n/\omega_1)\phi_1 \tag{46}$$

In addition, the phases may be defined relative a series of sine waves, while the ERM vibration equation Eq. 42 is specified in terms of a cosine wave. A cosine wave is a sinusoidal wave, but the phase is shifted by 90 degrees from a sine wave. Table I shows parameters for embodiments that superimpose 2, 3, and 4 sine waves. These parameters can be converted to relevant parameters for embodiments with 2, 3, and 4 ERM pairs, using Eq. 45 and Eq. 46 along with the 90-degree shift for the cosine representation. Table III, provided below, shows these parameters for ERM pairs which generate high levels of asymmetry. The method described in FIG. **52** can be used to specify parameters for any number of ERM pairs.

TABLE III

	Number of ERM Pairs		
	2	3	4
Pair 1 Amplitude: A1	1	1	1
Pair 1 Eccentricity	m_1r_1	m_1r_1	m_1r_1
Pair 1 Relative Phase ϕ_{r1} (degrees)	0	0	0
Pair 2 Amplitude: A2	0.5	0.71	0.81
Pair 2 Eccentricity	0.125 m_1r_1	0.1775 m_1r_1	0.2025 m_1r_1
Pair 2 Relative Phase ϕ_{r2} (degrees)	180	180	180
Pair 3 Amplitude: A3		0.33	0.54
Pair 3 Eccentricity		0.0367 m_1r_1	0.060 m_1r_1
Pair 3 Relative Phase ϕ_{r3} (degrees)		270	270
Pair 4 Amplitude: A4			0.25
Pair 4 Eccentricity			0.0156 m_1r_1
Pair 4 Relative Phase ϕ_{r4} (degrees)			0

Implementing General Synchronized Vibration with ERMs has an advantage that a wide range of vibration frequencies can be generated without being restricted to a specific resonance range. As the ERM frequency increases the centrifugal forces increase, the ratio of waveform amplitudes of A_1 and A_n remains constant. Accordingly, high levels of asymmetric vibrations can be generated with a single ratio of eccentricity, as shown in Table II and Table III, over an arbitrary frequency.

An embodiment with four ERMS is shown in FIG. **69**. ERMS **1222***a*, **1222***b*, **1224***a* and **1224***b* are stacked vertically inside a tube **1220**, which serves as the mounting platform **1202**. This embodiment could be used as a user input device which is grasped by the hand, similar to how the PlayStation® Move motion controller is grasped. Configurations with stacked ERMs are convenient for a wide range of hand held devices and to apply vibration forces to a wide range of body parts.

Steps of General Synchronized Vibration are shown in FIG. **70** for the case of a configuration shown in FIG. **69**. The parts shown in FIG. **69** are the same parts as shown in FIG. **70**, but part numbers are not called out in FIG. **70**. Each frame of FIG. **70** shows the eccentric masses of the ERMs and a line extending from each mass indicates the centrifugal force vector that the mass generates. The combined force vector of all ERMs is shown by the thicker line under the ERMs. In the embodiment shown in FIG. **70** the top two ERMs **1222***a* and **1224***b* are rotating clockwise from the top view perspective, and the bottom two ERMs **1222***a* and **1224***b* are rotating counter-clockwise. Furthermore the top **1224***b* and bottom **1224***a* ERMs have lower eccentric masses and are rotating at twice the frequency of the middle two ERMs **1222***a* and **1222***b*.

Other embodiments are possible with different frequency and mass relationships. The time, t, is represented in terms of the period of the fundamental frequency, where T1=2π/ω1. As seen in FIG. **70**, at time t=0, the forces of all ERMs are aligned with the axis of vibration in the positive direction, and the position of the eccentric masses are all aligned in the same orientation. Accordingly, at t=0 the combined vibration force has a large magnitude. At t=2T⅛ the combined force vector is in the negative direction along the axis of vibration, yet the negative magnitude is not at a peak value since contribution only occurs from ERM **1224***a* and ERM **1224***b*, while the forces from ERM **1222***a* and ERM **1222***b* cancel each other out. At t=4T⅛ the combined force vector is also in the negative direction along the axis of vibration, yet the negative magnitude is not at a peak value since there is negative interference between the first ERM pair (ERM **1222***a* and **1222***b*) and the second ERM pair (ERM **1224***a* and **1224***b*). At t=4T⅛ the forces of the first ERM pair are in the opposite direction of the forces from the second ERM pair, and the orientation of the eccentric masses of the first ERM pair is 180 degrees opposite the orientation of the eccentric masses of the second ERM pair. The magnitude of the combined vibration force is shown by the line beneath the eccentric masses at each point in time.

Another vibration device is shown in FIG. **71**, in which the device contains two ERMs **1230***a* and **1230***b* attached to a mounting platform **1202** that are rotating in the same direction. When the rotational speed and eccentricity of both ERMS are the same, this configuration is referred to as a Co-Rotating Pair, or "CORERM Pair". The center between the ERM eccentric masses is referred to as the center of the CORERM Pair. When the angle between the two ERMs is kept at a fixed value of angle, c, the CORERM Pair generates a combined centrifugal force that is equivalent to a single ERM. However, the magnitude of centrifugal force of the CORERM Pair is a function of the angle c. When c is equal to zero the combined force magnitude is twice that of a single ERM and when c is equal to 180 degrees then the centrifugal force magnitude is equal to zero since there is no overall eccentricity. Accordingly, when c is close to 180 degrees, the centrifugal force may not be the dominant force output of the CORERM Pair. Instead, gyroscopic or torque effects may take on a larger proportion of the force and torques applied onto the Vibration Device. Where A is the magnitude of force from just one of the ERMs in the pair, ω is the rotational speed, and ϕ is the phase of rotation, then the combined vibration force generated by a CORERM Pair is given by:

$$F_{CORERM} = 2A\cos(c)\begin{bmatrix} \cos(\omega t + \varphi) \\ \sin(\omega t + \varphi) \end{bmatrix} \quad (47)$$

A single vibration device could operate similar to ERMs as either counter-rotating pairs or co-rating pairs. There are a number of advantages of operating a vibration device in a mode where some of the ERMs function as CORERMs. One advantage is that the magnitude of vibration can be increased by using a CORERM pair. Another advantage is that legacy vibration effects can be generated that simulate a single ERM rotating. For example, a haptic interface could be operated at one time to generate asymmetric vibration forces and at another time to simulate a single ERM. If users are accustomed to haptic signals from a single ERM, the CORERM pair allows for such familiar effects to be generated.

A large number of co-rotating ERMs could by synchronized together in with no phase offset such that their force magnitudes combine to create a vibration effect similar to a single large ERM. If all the co-rotating ERMs are CORERM pairs with co-located centers, then the center for the combined force would be the same as for a single large ERM.

Another advantage of using CORERM pairs is that they allow for Fourier syntheses of a wider range waveforms. One such embodiment is to replace each ERM in FIG. **66** with a CORERM pair, which is shown in FIG. **72**. ERM **1212***a*, **1212***b*, **1214***a*, and **1214***b* in FIG. **66** correspond to CORERM **1232***a*, **1232***b*, **1234***a*, and **1234***b* in FIG. **72**. Such an embodiment would be similar to the original configuration of FIG. **66**, but where the magnitude of centrifugal force from each ERM could be adjusted independently of the speed of rotation (by adjusting the angle c within CORERM pairs). Fourier synthesis allows arbitrary waveforms to be approximated with a superposition of sine waves where the amplitude, phase, and frequency of the sine waves can be adjusted. With a sufficiently large number of CORERM pairs, any waveform with a zero-DC offset could be approximated. The embodiment in FIG. **72** also allows the direction of vibration to be controlled.

Control of amplitude of vibration force can be especially useful in asymmetric vibrations used for haptic applications. A vibration device can be grasped by one hand, two hands, held with other body parts, attached to any body part, or placed in contact with any body part. Generally at least two sides of a haptic vibration device are in contact with a user, and each side contacts the user at somewhat different locations on their body. These different locations could be the different sides of a grip of a tube vibration device, such as shown in FIG. **69**.

Human perception often requires that a threshold be exceeded before a sensory event is perceived. In one embodiment, the magnitude of an asymmetric waveform is adjusted so that on one side of a Vibration Device low vibration forces are generated that are below a threshold of perception and on the opposite side higher peak forces are generated that are above a threshold of perception. In this manner, a vibration force may be perceived on mostly one location that is in contact with the vibration device, even though the vibration device is in contact with a number of locations on the body. As the direction of vibration is varies, the location on the body at which vibration is perceived may also vary. This approach uses vibration to generate effects that are vary significantly according to the direction of vibration, and thus are useful for indicating directional cues.

An embodiment with 2 ERMs in a tube is shown in FIG. **73**. In FIG. **73**A the ERMs **1222***a* and **1222***b* are mounted close to the center of the tube **1220** and thereby reduce the torque vibration that is due to the distance between the ERMs. One way of controlling this configuration is to operate the ERMs in a counter-rotating mode and generate force in a specified direction, with only a small torque vibration so as to minimize distraction from the force effect. In FIG. **73**B the ERMs **1222***a* and **1222***b* are mounted close to the ends of the tube **1220** and thereby increase the torque vibration that is due to the distance between the ERMs. One way of controlling this configuration is to operate the ERMs in a counter-rotating mode and generate force in a specified direction, while simultaneously generating a large torque vibration effect.

The embodiment in FIG. **69** can also be operated with CORERM pairs. ERMs **1224***a* and **1224***b* can form one pair, and ERMs **1222***a* and **1222***b* can form another pair. When both of these CORERM pairs are rotating in the same direction and have a 180 phase difference, there will be no net force or net torque on the vibration device. However, this embodiment will create a gyroscopic effect with minimal force or torque vibrations. This implementation could be used to generate the sensation of moving a sword or a heavy mass in a video game or other type of simulation.

The forces between an ERM and a mounting platform include both centrifugal forces and the motor torque generated between the motor stator and rotor. When an ERM is rotating at operating speed, the centrifugal forces are typically large and dominate the effect from the motor torque. However, some embodiments can bring effects from the motor torque to the forefront. When two ERMs with parallel axes are operated as a co-rotating pair with a phase offset of 180 degrees, the two eccentric masses balance each other out and the centrifugal forces cancel each other out. In this embodiment, the torque about the axes of rotation can be felt more prominently. The torque about the axis of rotation is felt during the acceleration and deceleration of the rotating masses. Higher torques can often be generated by periodically reversing the applied voltage to the motor, since the electromagnetic force (back EMF) in the motor can add to the reverse voltage being applied.

Even higher torques about the axis of rotation can be generated by using a brake to cause a sudden deceleration to a rotating mass. This approach is known as a reaction-wheel method for generating torques, and is useful when there is no grounded actuator to apply a torque effect. FIG. **74** shows an eccentric mass **1206** configured for use as a reaction wheel. A rim **1242** is attached to the eccentric mass **1206**, and creates a surface for a brake **1244** to contact. When the brake **1244** is actuated a relatively high torque can be generated. The reaction-wheel configuration is another example of the wide range of effects that General Synchronized Vibration can generate. A single vibration device can have ERMs that are operated in counter-rotating modes, co-rotating modes, and as reaction-wheels.

One embodiment of an ERM Pair uses interleaved masses, an example of which is shown in FIG. **75**. In this embodiment, the shapes of the eccentric masses are implemented so that the masses can be interleaved within one another yet still rotate independently. With interleaved masses, both ERMs can share the same axis of rotation. In addition, a mass distribution can be implemented such that the eccentric forces share the same plane (which can be indicated by the height in the side view in FIG. **75**). Each ERM in the pair has a rotating mass that includes both an eccentric component, and a symmetric component such as the motor's rotor. The center of mass of the eccentric mass refers to the center of mass of only the eccentric component of the rotating mass. The center of mass of the eccentric mass rotates about the axis of rotation of the ERM, yet its position can be projected (in a linear algebra sense) onto a single point on the axis of rotation. With interleaved masses the geometry and density of the eccentric masses can be selected such that the center of mass of the eccentric masses from both ERMs are projected onto the same position on the axis of rotation. In this configuration the eccentric forces from both ERMs share the same plane. In FIG. **75**, ERM1 **1250***a*, contains a motor **1252***a* and an eccentric mass **1254***a* which is shaped with a semi-circle cross section, and ERM2 **1250***b*, contains a motor **1252***b* and an eccentric mass **1254***b* which is shaped with an arc cross-section. Other shapes of eccentric masses are possible that allow for independent rotation of two masses.

In an embodiment with interleaved masses, the ERM pair can generate centrifugal forces without generating a torque due to the distance between the ERMs. Interleaved ERM pairs are useful for generating pure force vibrations without torque vibrations. Interleaved ERM pairs can be operated as a co-rotating pair, and thereby vary the amplitude of vibration independently from the frequency of vibration. A co-rotating interleaved pair can switch between a 180 degree angle between the ERMs and a 0 degree angle to rapidly turn the vibration effect on or off. Since there are no torque effects, the complete vibration sensation will be turned off when the ERMs have a relative phase angle of 180 degrees. In addition, such a configuration can generate a gyroscopic effect without generating torque vibrations.

An interleaved ERM pair can also be operated as a counter-rotating pair, and thereby generate a vibration force along an axis. By controlling the phase of the interleaved ERMS, the direction of the vibration force can be controlled.

Embodiments with 3 ERMs are shown in FIGS. **76**A-B. In FIG. **76**A, a mounting platform **1202** shaped as a tube, holds a center ERM, **1312**, an ERM **1314***a* is located above the Center ERM, and an ERM **1314***b* is located below the center ERM. All 3 ERMs are aligned such that their axis of rotation is collinear. In this figure, the dimension A is the distance along the axis of rotation between the projection of the center of the rotating eccentric mass of ERM **1312** onto the axis of rotation and the projection of the center of the rotating eccentric mass of **1314***a* onto the axis of rotation. In a similar fashion, ERM **1314***b* is located such that it is at a distance B along the axis of rotation between the projection of the center of its eccentric mass onto the axis of rotation and that of the projection of the center of the rotating eccentric mass of ERM **1312** onto the axis of rotation. Furthermore, the ERMs **1314***a* and **1314***b* can be synchronized to operate at the same frequency and same phase, which will generate a combined force centered along the axis of rotation.

When the distance A times the eccentricity of ERM **1314***a* is equal to the distance B times the eccentricity of ERM **1314***b*, then the combined vibration force from synchronized ERMs **1314***a* and **1314***b* is projected onto the axis of rotation at the same point along this axis that the center of the eccentric mass of ERM **1312** is projected onto. In this configuration the combined vibration force from all 3 ERMs share the same plane. With this configuration, a vibration force can be generated by all 3 ERMs without generating a torque. Accordingly, the embodiment with 3 ERMs in FIG. **76**A can be operated in a mode where it is functionally similar to the embodiment with 2 ERMs shown in FIG. **75**, but the embodiment in FIG. **76**A uses standard shaped eccentric masses. The embodiment in FIG. **76**A can be operated in a co-rotation mode, where all 3 ERMs rotate in the same direction and with the same frequency. ERMs **1314***a* and **1314***b* can be operated with the same phase, and this phase can be adjusted relative to the phase of the center ERM, **1312**, which will modulated the amplitude of the vibration force.

If the eccentricity of ERM **1314***a* plus the eccentricity of ERM **1314***b* is equal to the eccentricity of ERM **1312**, then complete cancellation of the vibration forces can occur when all 3 ERMs are rotating. This complete cancellation allows for rapid on and off control of vibration forces. The embodiment in FIG. **76**A can also be operated in a counter-rotation mode, where the direction of rotation and phase of ERMs **1314***a* and **1314***b* are the same, yet the center ERM, **1312**, is operated in the opposite direction. In the counter-rotating mode, vibration forces along an axis can be generated, and the direction of the vibration can be controlled by modulation the relative phase of the ERMs. The embodiment in FIG. **76**A, also can be operated in a mode that is not similar to the interleaved embodiment in FIG. **75**; here, the center ERM can be turned off and ERM **1314***a* can be operated out of phase with ERM **1314***a* to create a rocking torque in the device. In addition, each ERM in FIG. **76**A can be operated at a different frequency. ERMs with smaller eccentric masses often can be operated at higher top frequencies, and thereby the embodiment in FIG. **76**A can create even a wider range of vibration effects.

Another embodiment with 3 ERMs is shown in FIG. **76**B. A mounting platform **1202** shaped as a tube holds a center ERM, **1312**, an ERM **1314***a* is located above the center ERM, and an ERM **1314***b* is located below the center ERM. All 3 ERMs are aligned such that their axis of rotation is collinear. In FIG. **76**B, the dimension A is the distance along the axis of rotation between the center of the rotating eccentric mass of ERM **1312** and the center of the rotating eccentric mass of **1314***a*. ERM **1314***b* is located at the same distance A along the axis of rotation between its center of the rotating eccentric mass and that of the center of the rotating eccentric mass of ERM **1312**.

When the eccentricity of ERMs **1314***a* and **1314***b* are half the eccentricity of the center ERM **1312**, and the ERMs **1314***a* and **1314***b* are synchronized to operate at the same frequency and same phase, then complete cancellation of vibration forces and torques can occur at a phase offset of 180 degrees. Thus, the embodiment in FIG. **76**B can have the same functional advantages as the embodiment in **76**A. A further advantage of the embodiment of FIG. **76**B is that two ERMs have identical specifications and thus can be more easily manufactured.

An additional embodiment with 3 ERMs is shown in FIG. **77**. A mounting platform **1202**, holds a center ERM, **1312**, an ERM **1314***a* is located to one side of the center ERM, and an ERM **1314***b* is located to the other side the Center ERM. All 3 ERMs are aligned such that their axes of rotation are parallel. When all 3 ERMs are rotating in the same direction, the embodiment in FIG. **77** can create similar vibration effects as the embodiments in FIGS. **76**A-B; the frequency off all 3 ERMs can be the same, the phase of ERMs **1314***a* and **1314**b can be the same, and the relative phase with the center ERM **1312** will determine the magnitude of the vibration force.

To provide complete cancellation of the vibration force, the eccentricity of the rotating mass of ERMs **1314**a and **1314**b can be selected to be half that of the center ERM **1312**. Complete cancellation of vibration torques can occur in the co-rotating mode when the center ERM **1312**, is located in the center between ERMs **1314**a and **1314**b. The embodiment in FIG. **77** can also be operated in a counter-rotating mode, where the ERMs **1314**a and **1314**b rotate in the same direction with the same phase, and the center ERM **1312** rotates in the opposite direction. This counter-rotating mode provides a vibration force along an axis, and the direction of the vibration force can be controlled by the phases of the ERMs. However, in the embodiment in FIG. **77**, there will be a vibration torque during the counter-rotating mode since the axes of the ERMs are not collinear.

The embodiment in FIG. **77** can also be operated in a counter-rotating mode, where the ERMs **1314**a and **1314**b rotate in the same direction with the same phase, and the Center ERM **1312** rotates in the opposite direction. This counter-rotating mode provides a vibration force along an axis, and the direction of the vibration force can be controlled by the phases of the ERMs. However, in the embodiment in FIG. **77**, there will be a vibration torque during the counter-rotating mode since the axes of the ERMs are not collinear.

General Synchronized Vibration of ERMS requires control of both the frequency and phase of rotating eccentric masses. One method is to use a motor, such as a stepper motor, where the position and speed can be defined open-loop by specifying a desired series of steps. Another method is to use closed loop control with a sensor or sensors that measure frequency and phase. An ERM with a sensor **1260** is shown in FIG. **78**. The sensor **1262** can be a continuous position sensor that measures the position of the eccentric mass at frequent intervals. Continuous sensors could be encoders, potentiometers, a Hall Effect sensor that detects a series of gear teeth or other feature of a rotating object, or other types of position sensors. The velocity of the eccentric mass could be calculated from the time interval between subsequent rotations, through taking the derivative of position measurements, or directly through use of a tachometer.

Another method to sense frequency and phase is to use a discrete sensor that detects when the motor shaft spins by a set position relative to the motor housing, or a number of set positions relative to the motor housing. Such discrete sensors can use reflective optical sensors that reflect off a rotating object coupled to the motor shaft, line-of-sight optical sensors that detect when a rotating object coupled to the motor shaft interrupts the line of site, hall effect sensors that detect a discrete component that is coupled to the rotating shaft, or other method of discrete detection of the shaft position.

FIG. **79** shows an ERM with a reflective optical sensor **1264** which detects light reflecting off an eccentric mass **1206**. A light source **1268**, such as an LED, is shining onto the pathway of the eccentric mass **1206**. When the eccentric mass **1206** rotates by the sensor **1266**, light reflects off the eccentric mass **1206** into the light sensor **1266**. For each rotation of the eccentric mass **1206** the light sensor **1266** will detect when the eccentric mass **1206** comes into the range of the sensor **1266** and begins to reflect light, and when the eccentric mass **1206** leaves the range of the sensor **1266** and stops to reflect light. The velocity of the ERM **1264** can be determined between the intervals of each rotation, such as the time when the eccentric mass **1206** begins to reflect light. Alternatively the velocity of the ERM **1264** can be calculated by the duration of time that the eccentric mass **206** reflects light. The phase of the eccentric mass **1206** can be determined by the timing of a specific event such as the rising or falling edge of the light sensor **1266** which corresponds to the time when the eccentric mass **1206** begins and stops reflecting light.

FIG. **80** shows an ERM with a line-of-sight optical sensor **1270**. The light sensor **1266** detects when the eccentric mass **1206** interrupts the light path. A light source **1268**, such as an LED, is shining onto the pathway of the eccentric mass **1206**. When the eccentric mass **1206** rotates through the light path, the sensor **1266** detects the interruption. FIG. **81** shows an ERM with a Hall Effect sensor **1272**. The Hall Effect sensor **1274** is triggered when the eccentric mass **1206** rotates by.

Implementing General Synchronized Vibration with ERMs requires that the frequency and phase be controlled for each ERM that is used to synthesize the desired waveform. Both the frequency, ω, and phase, ϕ, can be controlled by controlling the position, θ, of the rotating shaft of the ERM to be at a desired position as a function of time. Accordingly, control of frequency and phase can also be equivalent to control of the position of an eccentric mass to a desired position trajectory over time. Measurement of the shaft position can be performed continuously or at discrete instances such as when the shaft passes a certain position. Continuous measurements could be made with an encoder or other type of sensor that measures positions at frequency intervals. Discrete measurements could be made with an optical sensor that detects when the eccentric mass passes by. Discrete measurements could be made at a single position of motor rotation or at multiple positions. Discrete measurements can be augmented with a second sensor that also measures the direction of rotation. A direction sensor could be a second optical sensor mounted close to the first optical sensor. The direction of rotation can be determined by which optical sensor is triggered first.

A wide range of methods can be used for real-time control the position and speed of an ERM. One method is Proportional-Integral-Control. Another method includes time optimal control as described by "Optimal Control Theory: An Introduction", by Donald E. Kirk, Dover Publications 2004. One real-time control approach is presented below for controlling a set of synchronized ERMs. The approach is written for use with a discrete sensor, but can also be applied with a continuous sensor. When a continuous sensor is used, the dynamic performance of the system can be improved by more accurately updating the commands to the motor continuously.

An exemplary control approach for a system with M ERMs is now discussed. For each ERM for i=1 to M, define the desired frequency, ωdes,i, and desired phase, $\phi_{des,i}$. The desired direction of rotation is defined as $dir_{des,i}=\mathrm{sign}(\omega_{des,i})$. Initialize the following variables:

a. Time, t=0

b. Number of revolutions of each ERM, $nrev_i=0$ (for all i)

Next, start motors turning by providing an open-loop command, $V_{open\_loop,i}$, to each ERM corresponding to the desired frequency, $\omega_{des,i}$. The open-loop command can be determined by the motor's torque-speed curve and correspond to the voltage that will generate a terminal velocity as the desired value. An optional startup operation is to turn on the motors at a high or maximum voltage to reduce the startup time. Since sensors exist to detect speed of rotation, the voltage can be reduced to a desired level when the ERMs reach an appropriate speed. In this fashion the sensors used for synchronization can also be used to reduce the startup time of the overall vibration device. As each ERM passes its discrete sensor:

c. Measure the time and record: $t_{meas,i}=t$

d. Calculate the desired position at the measured time:

$$\theta_{des,i}=\omega_{des,i}t_{meas,i}+\phi_{des,i}$$

e. Calculate the measured position, $\theta_{meas,i}$, at the measured time:

Increment the number of revolutions: $nrev_i=nrev_i+1$

$$\theta_{meas,i}=2\pi dir_i nrev_i+\theta_{sensor\_offset,i}$$

$\theta_{sensor\_offset,i}$ is based upon the mounting location of the discrete sensor, and is often equal to zero.

dir_i is the actual direction of the ERM rotation. Typically the ERM will be rotating in the direction of the initial open-loop command. However, it is also possible to use a second sensor input to measure the direction of rotation, or use the time history of the motor command to calculate the direction.

f. Calculate the error in position, $\theta_{error,i}$, for each ERM:

$$\theta_{error,i}=\theta_{des,i}-\theta_{meas,i}$$

A control law may be implemented to reduce the position error of each ERM. There are a wide range of control of control approaches in the field of control, including:

g. Proportional, Integral, Derivative ("PID") based upon the calculated error in position. The command to the motor would be:

$$V_{com,i}=K_{p,i}\theta_{error,i}+K_{I,i}\int\theta_{error,i}dt+K_{D,i}d\theta_{error,i}/dt$$

h. Use the open-loop command as a baseline command to the ERM, since it is based upon the motor's characteristics, and apply PID to correct for remaining errors. The command to the motor would be:

$$V_{com,i}=V_{open\_loop,i}+K_{p,i}\theta_{error,i}+K_{I,i}\int\theta_{error,i}dt+K_{D,i}d\theta_{error,i}/dt$$

The use of the open-loop command can reduce the need for a large integral control gain, and improve dynamic performance.

i. State-space control approach. The physical state of each ERM is a function of both its position and velocity. Each time an ERM passes its discrete sensor, the speed of revolution can be calculated from the time interval since the last sensor measurement. The state-space approach uses both the position and velocity to determine an appropriate control signal. For the durations where no sensor measurements are made, a state observer can be used to estimate the motor's position and speed, where the model of the state observer is based upon the physical properties of the motor and rotating mass.

j. Use bang-bang control, which operates the motor at maximum forward command and maximum reverse commands for specified durations of time. For example, if an ERM is operating at the correct speed but position has a phase lag, then the motor should be accelerated for a duration of time and then decelerated back to the original speed for a second duration of time. A physical model of the motor dynamics can be used to determine the appropriate durations of acceleration and deceleration.

With all control approaches a bidirectional or unidirectional motor driver could be used. An advantage of using bidirectional motor drivers is that high levels of deceleration can be applied to an ERM by applying a reverse voltage, even if the motor never changes direction of rotation. This approach can reduce the time it takes to synchronize the ERMs. Another advantage of using bidirectional motor drivers is that ERMs could be operated in both counter-rotating and co-rotating modes.

An alternative method of calculating the position error is discussed below. Where the desired force is represented by $A_i\sin(\omega_i t+\phi_i)$ and the desired position is represented by $\theta_1(t)=\omega_i t+\phi_1$, start all ERMs at open loop voltages corresponding to ω_1. Let the motors spin up to speed when ERM 1 passes the sensor so that it starts in phase, then reset the timer so t=0. See Table IV below.

TABLE IV

Control of ERM

Time at which sensor is triggered	θ measured (θ_{meas})	θ desired (θ des)	Δ t	θ desired (θ_{des})	Change in θdes (Δθdes)	$\Delta\theta_{meas}$	θ_{error}
t1	0	$\omega t_1+\phi$					$\theta_{des}-\theta_{meas}$
t2	2π	$\omega t_2+\phi$	t_2-t_1	$\omega\Delta t+\theta_{des\_prev}$		2π	$\theta_{error}=$ $\theta_{error\_prev}+\Delta\theta_{des}-\Delta\theta_{meas}$
t3	4π	$\omega t_2+\phi$	t_3-t_2	$\omega\Delta t+\theta_{des\_prev}$	$\theta_{des}-\theta_{des\_prev}$	2π	

In a digital system, ERM control may include the following. First, set rotation counts per revolution (e.g., 256 or 512). Correct for timer overflow so $\Delta t=t_1-t_{i-1}$ is always correct. Define ω in terms of rotation counts per timer counts. And use interrupts (or other operations) to avoid missing when an ERM passes by a sensor.

Some embodiments of Synchronized Vibration Devices can be controlled such that the combined force and torque sum to zero. In such an embodiment the force and torques from individual Vibration Actuators balance each other out to generate a net zero force and torque. An advantage of such an embodiment is that Vibration Actuators can be brought up to speed and put into a mode when no vibration effects are generated. When vibration effects are desired, they can be quickly implemented by modifying the phase of the vibration, without the lag for bringing the actuators up to speed. This embodiment is referred to as "Spinning Reserve", and is analogous to the same term used for kinetic energy in an electric utility power plant that is held in reserve to quickly provide power when needed. The spinning reserve approach allows vibration to be quickly turned on and off. Spinning

Reserve embodiments can include ERM actuators that are spinning in such a manner that the combined forces and torque sum to zero. Spinning Reserve embodiments can also include with LRA actuators and other resonant actuators that are vibrating in such a manner that the combined forces and torque sum to zero.

The spinning reserve approach has the advantage of fast on and off response times, but also can require increased power consumption since the vibration actuators are operated even when no overall vibration effects are generated. To reduce the added power consumption, the vibration actuators can be spun up to speed at the first indication that a need for vibration force is imminent. Such indications could be a keystroke, computer mouse motion, user touching a touchscreen, movement detection via a sensor of a game controller, beginning of a game portion where vibration effects are used, or any other event that would indicate that a desired vibration effect would be imminent. In a similar fashion power can be conserved by spinning down and stopping the actuators once the need for vibration is no longer imminent. Indications to spin down the actuators could include passage of a set amount of time where no user input is registered, transition to a new phase of a computer program where vibration effects are no longer needed, or other indication. During the spin up and spin down of the actuators, the actuators can by synchronized so that they operate in a spinning reserve mode and do not generate a combined vibration force. In this fashion, the user will not feel the spin up and spin down of the vibration actuators.

A spinning reserve embodiment with 4 ERMs is shown in FIG. **82**. Synchronized Vibration can be applied to the embodiment shown in FIG. **82**, where the combined forces and torque cancel each other out. In one such embodiment the eccentricity and rotational inertia of ERMs **1190***a*, **1190***b*, **1192***a* and **1192***b* are equal to each other. In one such control method all 4 ERMs rotate in the same direction. The frequency and phase could be as shown in Table V below. The synchronized phases within a set of ERMs, can be controlled relative to each other and not just relative to absolute time. Accordingly, the phases shown in Table V and other tables in this document only represent one set of phases in absolute time that achieve the described effect. Other phase combinations can achieve similar effects.

TABLE V

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	ω	ω	ω	ω
Phase	−90°	90°	90°	−90°

FIG. **83** shows the forces of the ERMs from Table V as the ERMs progress through time, wherein each row of images illustrates one time slice (8 slices in all). The parameters for frequency and phase shown in Table V correspond to the force vectors shown in FIG. **83**. In a similar fashion other configurations and control methods of vibrations devices can also be simulated.

Another method of Synchronized Vibration can be applied to the embodiment shown in FIG. **82**, where the combined forces and torque cancel each other out. In such a control method ERM **1190***a* rotates in the opposite direction of ERM **1190***b*, and ERM **1192***a* rotates in the opposite direction of ERM **1192***b*. The frequency and phase could be as shown in Table VI.

TABLE VI

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	ω	ω	−ω	−ω
Phase	−90°	90°	90°	−90°

When ERMs are rotating they generate a gyroscopic effect due to the angular inertia of the motor rotor and rotating mass. When the angular velocity of the ERMs is large this gyroscopic effect can be used to generate a haptic sensation in response to changes in orientation of the vibration device. The implementation of spinning reserve as shown in Table V has a gyroscopic effect since all ERMs are rotating in the same direction and their angular inertia combined. The implementation of spinning reserve as shown in Table VI does not have a gyroscopic effect since half the ERMs are rotating in the opposite direction of the other half, and therefore angular inertias cancel each other out when rotational inertias are equal. The mode of implementation of spinning reserve can be selected according to the desired gyroscopic effect.

Another method of Synchronized Vibration can be applied to the embodiment shown in FIG. **82**, where the combined forces generate a force along the x axis and the torques cancel each other out. The frequency and phase could be as shown in Table VII.

TABLE VII

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	ω	−ω	−ω	ω
Phase	0°	0°	0°	0°

Another method of Synchronized Vibration can be applied to the embodiment shown in FIG. **82**, where the combined forces generate a force along the y axis and the torques cancel each other out. The frequency and phase could be as shown in Table VIII.

TABLE VIII

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	ω	−ω	−ω	ω
Phase	90°	90°	90°	90°

Indeed Synchronized Vibration can be applied to the embodiment shown in FIG. **82**, where the combined forces generate a force along any axis in the XY plane. The control that implements an axis at 30 degree and the torques cancel each other out is shown in Table IX.

TABLE IX

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	ω	−ω	−ω	ω
Phase	30°	30°	30°	30°

Another method of Synchronized Vibration can be applied to the embodiment shown in FIG. **82**, where a combined torque is generated and the forces cancel each other out. One such pure torque embodiment generates equal amplitudes torque in the clockwise and counterclockwise directions, and is referred to as a symmetric torque implementation. The frequency and phase that generates a symmetric torque could be as shown in Table X.

TABLE X

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	−ω	ω	−ω	ω
Phase	−90°	−90°	90°	90°

Another implementation of pure torque can produce an asymmetric torque, where the peak torque in the clockwise direction is larger than the peak torque in the counterclockwise direction, or vice versa. One such asymmetric torque implementation for a 4 ERM configuration could be as shown in Table XI. This is achieved by operating ERMs **1192***a* and **1192***b* at twice the frequency of ERMs **1190***a* and **1190***b*, and controlling the phase appropriately. For the configuration shown in FIG. **82**, when all ERMs have the same eccentricity, the amount of asymmetry in the torque can be increased by placing ERMs **1192***a* and **1192***b* at a distance of ⅛th from the center relative to the distances of ERMs **1190***a* and **1190***b*.

TABLE XI

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	ω	2ω	2ω	ω
Phase	−90°	−90°	90°	90°

Yet another method of Synchronized Vibration can be applied to the embodiment shown in FIG. **82**, where all ERMs rotate together and forces do not cancel each other out. This implementation generates an effect of one large ERM that would have the eccentricity of all ERMs combined. The frequency and phase that generates a symmetric torque could be as shown in Table XII.

TABLE XII

	ERM 1190a	ERM 1192a	ERM 1192b	ERM 1190b
Frequency	ω	ω	ω	ω
Phase	0°	0°	0°	0°

A wide range of haptic effects can be generated by switching between the various effects described herein. When the ERMs are rotating at the same speed in two different effects, the change between effects (including the no-vibration spinning reserve) can be achieved quickly. In many cases the change in effect only requires a positive or negative phase change of 90 degrees in specific ERMs.

Embodiments with 4 ERMs that are not aligned along the same axis also can generate many useful effects. FIG. **84** shows an embodiment of 4 ERMs. When this embodiment is implemented with 4 ERMs with the same eccentricity, a spinning reserve effect can be generated with same frequency and phases shown in Table V. In FIG. **84** the center of ERM pair **1194***a* and **1194***b*, has the same center as ERM pair **1196***a* and **1196***b*. Indeed, any embodiment with 2 pairs of ERMs that share the same center can be controlled in a spinning reserve mode.

The embodiment shown in FIG. **84** can also be controlled to generate a pure force vibration along a specified direction, where the torques cancel each other out. The same frequency and phase as shown in Table VII, Table VIII, and Table IX can be used. A symmetric torque can be generated with this embodiment as well, but with a frequency and phase as defined in Table V, and replacing ERMs **1190***a*, **1190***b*, **1192***a*, and **1192***b* with ERMs **1194***a*, **1194***b*, **1196***a*, and **1196***b*, respectively.

As discussed above with regard to FIG. **75**, interleaved ERM pairs may be employed according to aspects of the disclosure. Another embodiment of an interleaved ERM pair is shown in FIGS. **85**A-B. As shown in FIG. **85**A, an inner eccentric mass **1320***a* is driven by motor **1322***a* and an outer eccentric mass **1320***b* is driven by motor **1322***b*. The outer eccentric mass **1320***b* is shaped so that the walls get thicker going away from the motor **1322***b*. This extra thickness compensates for the material required for structural support of the eccentric mass near the motor. As shown in the side view of FIG. **85**B, the inner eccentric mass **1320***a* fills the void inside eccentric mass **1320***b*. The result is that both eccentric masses **1320***a* and **1320***b* share the identical center of mass, which eliminates unwanted torque effects.

Another embodiment of an interleaved ERM pair is shown in FIGS. **86**A-C. Here, an inner eccentric mass **1330***a* is driven by motor **1332***a* and an outer eccentric mass **1330***b* is driven by motor **1332***b*. The end of eccentric mass **1330***a* that is furthest from the motor **1332***a* is supported by a bearing **1334***b*, which is installed into eccentric mass **1330***b*. The end of eccentric mass **1330***b* that is furthest from the motor **1332***b* is supported by a bearing **1334***a*, which is installed into eccentric mass **1330***a*. The bearings **1334***a* and **1334***b* allow for the spinning eccentric masses **1330***a* and **1330***b* to be supported on both ends. This allows the eccentric masses **1330***a* and **1330***b* to spin faster without deflection due to cantilever loads, and helps reduce friction in the motors **1330***a* and **1330***b*.

The performance of almost any vibration device can be improved by applying the methods and embodiments of General Synchronized Vibration discussed herein. This approach toward synchronization allows for a wide range of waveforms to be generated including asymmetric waveforms that generate larger peak forces in one direction than the opposing direction. Applications range from seismic shakers and fruit tree harvesters, to vibratory feeders and miniature vibration applications. The embodiments described herein can replace more expensive actuation devices that are used to generate complex waveforms of vibrations. Such applications include seismic shakers that are simulating specific earthquake profiles, and voice coils that are used to generate complex haptic effects.

Haptic applications described herein can be used to augment any device that has a visual display including computer gaming, television including 3D television, a handheld entertainment system, a smartphone, a desktop computer, a tablet computer, a medical device, a surgical instrument, an endoscope, a heads-up display, and a wristwatch. Implementation of haptic feedback within a system that has a visual display is shown in FIG. **87**.

As described herein, Vibration Force cues can be generated in specific directions, and these directions can be chosen to correspond to direction that is relevant to an object or event that is being shown on a graphic display. FIG. **88** shows a graphic display with an image that has a direction of interest specific by an angle σ. The Vibration Device shown in FIG. **88** can generate haptic cues in the same direction to provide multi-sensory input and enhance the overall user experience.

Moreover, it is be useful to generate haptic cues of directionality for applications where a person does not have visual cues, such as to guide a blind person or applications where vision is obscured or preoccupied with another task. For example, if a person had a handheld device such as a mobile phone that could generate directional haptic cues through vibration, and the mobile phone knew its absolute orientation as it was being held and the orientation the person should be in to move forward to a goal, then the mobile phone could communicate directional haptic cues through vibration (a force, a torque, or a combined force and torque) that corresponded to the direction and magnitude of the change in orientation the person holding the mobile phone needed to make.

The Vibration Devices describe herein can be used to improve the performance of existing devices that use vibration. For example vibration is used in fruit tree harvesting. By allowing the operator to generate complex waveforms and control the direction of vibration a higher yield of ripe fruit could be harvested, while leaving unripe fruit on the tree. Vibratory feeders are used in factory automation, and typically involve a significant amount of trial an error to achieve the desired motion of the parts. By allowing the operator to generate complex waveforms and control the direction of vibration it can be easier to generate the desired part motion and a wider range of parts could be processed with vibratory feeders.

The Vibration Devices described herein allow for a wide and continuous adjustment in areas such as vibration magnitude, frequency, and direction. To improve performance of a Vibration Device, sensor feedback can be used, as shown in FIG. **89**. With this approach a Vibration Device applies forces onto an object, and a sensor measures a feature or features of the object. The sensor information is provided to the Vibration Device Controller, which can then modify the vibration waveform to improve overall system performance. One area of application could be a vibratory parts feeder, where a sensor measures the rate at which parts move along a pathway, and the waveform is modified to improve the part motion. Another area of application could be preparation and mixing of biological and chemical solutions. A sensor could measure the effectiveness of the mixing and the vibration waveforms could be adjusted accordingly.

One application is to use General Synchronized Vibration for locomotion. FIG. **90** shows an embodiment where a Vibration Device **1200** rests on a surface **1282**. There exists friction between the surface **1282** and the Vibration Device **1200**. Accordingly, motion of the Vibration Device **1200** will only occur if a force parallel to the surface **1282** exceeds a friction threshold. In this embodiment, an asymmetric waveform is being generated so that the peak positive force exceeds the friction threshold and the peak negative force is less than the friction threshold. Accordingly in each vibration cycle the Vibration Device **1200** can be pushed in the positive x direction when the peak force in the positive x direction exceeds the friction threshold.

However, there will generally be no motion in the negative x direction, since the friction threshold is not exceeded. In this fashion, the Vibration Device **1200** will take steps in the positive x direction. The direction of motion along the x axis can be reversed by changing the synchronization of the Vibration Actuators and generating an asymmetric waveform that has a larger peak force in the negative direction. A location device can be made to move in arbitrary directions on a surface **1282** by using a Vibration Device **1200** where the direction of vibration can be controlled on a plane, such as those shown in FIG. **62** and FIG. **66**. In a similar fashion a locomotion device can be made to rotate by generating asymmetric torque vibrations, such as the one shown in FIG. **57**.

Vibration is also used for personal pleasure products such as Jimmyjane's Form 2 Waterproof Rechargeable Vibrator. Vibration is also used for personal massager products such as the HoMedics® Octo-Node™ Mini Massager. Vibration is also used for beauty products such as Estée Lauder's TurboLash and Lancôme's Ôscillation mascara applicators. INOVA produces the AHV-IV Series Vibrators for Vibroseis seismic exploration. General Synchronized Vibration can be used to improve the performance of such products by allowing the user to customize the vibration waveforms and direction of peak vibration forces.

General Synchronized Vibration may also be used in therapeutic medical applications. For example a Vibration Device could vibrate a patient's stomach to aid in digestion, and the patient or a sensor could determine how to adjust the vibration over time.

Now having developed a foundation of General Synchronized Vibration in the previous section, the architecture for a "Synchronized Array of Vibration Actuators in a Network Topology" (hereinafter "SAVANT") is now presented to motivate the use of multiple, low-cost actuator components in lieu of a single, high-cost actuator. When using multiple actuators in an array, the system is able to exhibit various modalities which we will refer to as dimensions of the SAVANT architecture. These orthogonal dimensions are: performance, bandwidth, reliability, magnitude, spatial and temporal.

The SAVANT architecture is generically applicable to any type of low-cost actuator, but for the purposes of the present analysis, systems of linear resonant actuators (LRAs) are discussed. It will first be shown that the analysis for multiple, collinear LRAs can be reduced to the analysis of a single LRA driven by the sum of the component LRA forcing functions. Then, example implementations of the different dimensions of SAVANT will be presented and compared with different single-actuator solutions currently available. Finally a discussion of multi-dimensional control effects and sequences of these effects will be presented.

I. Introduction of SAVANT Architecture

Having developed a foundation of General Synchronized Vibration in the previous section, a new conceptual framework is introduced for creating haptic effects with vibration actuators. This architecture is designed primarily with small, hand-held consumer devices in mind but it is general enough to apply to any device of any size. The basis of the architecture is joining small actuator elements together in different physical geometric configurations, or network topologies, to create a Synchronized Array of Vibration Actuators in a Network Topology—or SAVANT.

The power of SAVANT is three-fold: to synchronize arrays of low-cost, readily available vibration actuators to emulate superlative single actuators; to bring together sets of these emulated high-performance actuators to create almost any desired control effect; and to have an array of vibration actuators that is fault tolerant.

Any array of rigidly or semi-rigidly coupled actuators whose motions and control schemes have been synchronized with the purpose of having characteristics or producing haptic effects beyond the abilities of any single actuator in the array is a SAVANT. Subsets of the set of actuators in a SAVANT are also called SAVANTs as long as they still meet the definition.

While the number of actuators in a SAVANT must be at least two, SAVANTs may be further decomposed into a set of connected SAVANT nodes such that the number of actuators in a SAVANT node must only be at least one. The advantage of representing a SAVANT as a network of SAVANT nodes is that it enables the specification of geometric relationships between the SAVANT nodes. The specification of a SAVANT's nodes as a particular geometrical configuration is defined to be that SAVANT's specific network topology.

Furthermore, the SAVANT node is further defined to be a collection of rigidly or semi-rigidly coupled actuators in close proximity of one another whose resultant vibration effects are designed to appear to emanate from a single spatial point. These groups of actuators are also referred to herein as proximal groups. Groups of actuators that are spatially separated so as to jointly create effects that require non-pointlike origin are referred to herein as distal groups or distal nodes.

The network topology of the actuators in a SAVANT determines how they can be used together. The relevant properties of the SAVANT are the collinearities of the component actuators, the spanning set of displacement vectors and the relative proximities of the nodes.

The division of an array into SAVANT subsets is fluid. A single device may contain a SAVANT having a plurality of synchronized actuators and at any given time these actuators may be functioning in any available capacity. For example a device could have a total of six synchronized actuators: in response to stimulus A the six actuators could be divided into three subsets of two-actuator SAVANTs to produce the resultant control effect α; in response to stimulus B, the six actuators could instead be divided into two subsets of three-actuator SAVANTs to produce the control effect β. By joining together low-cost component actuators to emulate a single actuator with arbitrarily superlative performance characteristics, one also gets haptic capabilities unavailable to devices with unsynchronized vibration actuators, such as directional vibration and asymmetric vibration.

An LRA-type actuator has three performance characteristics that can be improved by combining multiple actuators together: response time, bandwidth and force output. An array of collinear LRA-type actuators can emulate improvement of any of these separately or at the same time. For example, four collinear LRAs could have synchronized control schemes such that they emulate a single component LRA with a faster response time. Alternatively they could be controlled synchronously to emulate a single component LRA with an increased frequency response range, or bandwidth. Or in a completely separate scenario the same four LRAs could be split into two subsets where each subset of two LRAs emulates a single LRA with increased bandwidth and collectively the two subsets work together to emulate faster response times.

Since an array of actuators with a given network topology can work together to emulate any single improved performance characteristic without improving the other two, then in the space of the actuator control schemes we consider these performance modes to be orthogonal. Thus it is possible for a SAVANT to simultaneously be in multiple performance modes, but it is not necessary. Later we will consider many examples of these control modes as well as combinations of them.

Beyond out-performing single actuator elements, arrays of actuators also have available two control modes unique to multiple synchronized actuators. These modes are referred to as spatial and temporal. SAVANTs running in spatial mode can create haptic effects that relate to the user via their position and orientation in space. For instance a set of actuators may work together to create a half-wavelength oscillation or the amplitude of a vibrational effect may change based on the position of the device (per se relatively to another object or positions related to the Earth). In temporal mode, SAVANTs can create vibrational effects that interact with the user to create an awareness of time. These can include asymmetric waveforms created through Fourier synthesis of harmonic forcing functions.

Finally, proximal arrays of collinear actuators have a natural safeguard against individual component failure. Many devices will use haptic effects to protect the wellbeing of their users, often in perilous or extreme environments. It is crucial that the vibration actuators are fault tolerant. One of the ways to achieve this is to build in redundancies for the purposes of reliability. Arrays of actuators that are designed to work together for the purposes of fault tolerant reliability are SAVANTs said to be running in reliability mode.

Because SAVANTs can exhibit these control modalities independently or simultaneously, they may be combined together into a single vector space which is referred to herein as the SAVANT "Control Space." This space spans the haptic capabilities of sets and/or subsets of actuators. The purpose of introducing this space and its comprising directions is to abstract the capabilities of groups of actuators. The discussion of haptic control schemes can be elevated from "on and off" to sequences of control effects designed to enhance the user experience beyond today's capabilities, while only using low-cost components that are readily available today.

All possible haptic effects (within reason) can be created with the SAVANT architecture. In general, a SAVANT of sufficient size can be thought of as a vibration synthesizer, designed to create arbitrary vibrational output either using predefined control sequences or in response to external information, e.g., user interface or sensor feedback. Generically the SAVANT architecture applies to any type of actuator—and SAVANTs of one actuator type can even emulate another actuator type. For instance, two LRA-type actuators can be synchronized to emulate the output of an ERM-type actuator and vice versa. Given this duality, all control sequences designed e.g. for an LRA-type SAVANT are equally valid for an ERM-type SAVANT where each LRA-type actuator is replaced with two synchronized ERMs designed to emulate an LRA.

As previously stated, for the purpose of this inventive disclosure the examples of specific electronic components are designed for haptic applications of hand-held devices. The SAVANT architecture is not limited to actuators of this dimensional scale. That is to say, SAVANT is intended to be applicable at other dimensional scales with appropriate actuators—for example, an array of MEMS LRA actuators which might use electrostatic forces in place of electromagnetic forces; or arrays of very large LRAs that may be used for seismic exploration. Moreover, the examples discussed herein are based on a relatively small number of actuators but the architecture is consistent for any number of actuators. And finally, though the majority of examples in this text are concerned with homogeneous SAVANTs, it is perfectly acceptable and often desirable to have heterogeneous arrays of actuators. Here the notion of heterogeneity includes both actuators of a similar type with varying characteristics and actuators of different types.

FIG. **91** is a diagram illustrating the six dimensions of the SAVANT Control Space: Bandwidth, Magnitude, Performance, Reliability, Spatial and Temporal.

FIG. **92**A illustrates an example of a SAVANT node having a single LRA, **1102***a*. FIGS. **92**A-**92**J illustrate an example LRA that has a coin-like shape and has an axis of vibration normal to the surface of the LRA. As described previously, the minimum number of actuators that a SAVANT node must have is one. The minimum number of actuators that a SAVANT must have is two. Thus it follows, for example, that a 2-SAVANT (that is to say, a SAVANT having two actuators) may have a network topology of either one SAVANT node having two actuators or alternatively two

SAVANT nodes each having one actuator. Although an LRA is used in this example, other actuators may be used as long as they are simple harmonic systems, or can be combined or controlled to behave as simple harmonic systems. Some alternatives to LRAs include the various actuator types discussed above.

FIG. **92**B illustrates an example of a SAVANT node having two LRAs, **1102***a* and **1102***b*, arranged in a stack, with their axes of vibration vertically aligned.

Correspondingly, FIG. **92**C illustrates an example of a SAVANT node having three LRAs, **1102***a*, **1102***b* and **1102***c*, arranged in a stack, with their axes of vibration vertically aligned.

FIG. **92**D illustrates an example of a SAVANT node having two LRAs, **1102***a* and **1102***b*, in a compact planar arrangement, with their axes of vibration vertically aligned in parallel.

Correspondingly, FIG. **92**E illustrates an example of a SAVANT node having three LRAs, **1102***a*, **1102***b* and **1102***c*, in a compact planar arrangement, with their axes of vibration vertically aligned in parallel.

FIG. **92**F illustrates an example of a SAVANT node having three LRAs, **1102***a*, **1102***b* and **1102***c*, in a compact arrangement with their axes of vibration spanning three dimensions.

FIG. **92**G illustrates a side view (left) and a perspective view (right) of an example of a SAVANT node having three LRAs, **1102***a*, **1102***b* and **1102***c*, in a arrangement around three faces of a cube, **2000**, with their axes of vibration spanning three dimensions.

Correspondingly, FIG. **92**H illustrates a side view (left) and a perspective view (right) of an example of a SAVANT node having six LRAs, **1102***a*, **1102***b*, **1102***c*, **1102***d*, **1102***e* and **1102***f*, in a arrangement around the six faces of a cube, **2000**, with their axes of vibration spanning three dimensions.

Correspondingly, FIG. **92**I illustrates a side view (left) and a perspective view (right) of an example of a SAVANT node having twelve LRAs in a arrangement around the six faces of a cube with their axes of vibration spanning three dimensions.

FIG. **92**J illustrates a side view (left) and a perspective view (right) of an example of a SAVANT node having four LRAs in a tetrahedral arrangement around the four faces of a tetrahedron, **2004**, with their axes of vibration spanning three dimensions in accordance with aspects of the present disclosure.

FIG. **93** illustrates the equivalence between an exact representation of the vibration actuators in a SAVANT, **2020**, and a triangularly shaped symbol, **2022**, symbolizing a SAVANT node. In accordance with the nomenclature herein, this SAVANT node symbol may represent any of the previous examples of a SAVANT node, as illustrated in the above FIGS. **92**A-**92**J, or any types of actuators and arrangements that meet the requirements of a SAVANT node. For example, FIGS. **85**A, **85**B, and **85**C illustrate a vibration device having a pair of ERMs. This pair of ERMs can be considered both a SAVANT node and a SAVANT, since it meets the requirements for each.

FIG. **94** illustrates a SAVANT node, **2022**, inside an example smartphone or PDA, **2024**. In this example, the SAVANT node, **2022** may be attached to the smartphone case and disposed directly under a floating touch display, **2026**, such that the floating touch display is capable of vibrating vertically, when the SAVANT node, **2022**, is controlled by a controller in the smartphone, **2024**, to produce haptic sensations to a user in accordance with aspects of the present disclosure. In this manner, the SAVANT node, **2022**, can emulate a virtual actuator that can produce the haptic sensation that a person is pressing a physical button when in fact the person is only pressing the floating screen vibrated by the SAVANT node, **2022**.

FIG. **95** illustrates a SAVANT, **2034**, having twelve nodes in a three by four arrangement, **2022**, inside an example tablet computer, **2030**. In this example, the SAVANT, **2034**, is attached to the tablet computer case and disposed directly under a floating touch display, **2032**, such that the floating touch display is capable of vibrating vertically, when the SAVANT, **2034**, is controlled by a controller in the tablet computer, **2030**, to produce haptic sensations to a user in accordance with aspects of the present disclosure. In this manner, the SAVANT, **2034**, can emulate a set of virtual actuators that can produce the haptic sensation that a person is pressing a physical button when in fact the person is only pressing the floating screen vibrated by the SAVANT, **2034**. Since each of the twelve SAVANT nodes in this example may be individually or collectively controlled, various methods to produce localized haptic effects for a multi-touch display are possible.

Finally, FIG. **96** illustrates a perspective view (top) and a front view (bottom) of an example game controller, **2012**, having a module, **2012**, with a SAVANT inside. In this example, the SAVANT, not shown, is attached inside the module, **2012**, to impart vibrations to the game controller to enhance the realism of gameplay through haptic cues and other haptic sensations, including directional vibrations in accordance with the present disclosure.

II. Example Implementations of SAVANT Using LRA Arrays

A. Equation of Motion for an LRA Array

First consider a single LRA. The response function of an LRA is that of a damped harmonic oscillator with a resonant angular frequency ω_0, a mass m and a damping ratio ζ. Usually, though, LRAs are labeled by their resonant frequency f_0, where $\omega_0=2\pi f_0$. Generically, the LRA can be driven with any arbitrary function of time—usually implemented via the methods of pulse-width modulation, or PWM—but for simplicity let us just consider a simple sinusoidal driving force. This simplification is well-motivated by hardware considerations and as shown below, it is still quite general. Thus the equation of motion for a single LRA will be the solution of the differential equation

$$x''(t)+2\zeta\omega_0 x'(t)+\omega_0^2 x(t)=\frac{f\sin(\omega t+\phi)}{m} \quad \text{(Eq. 101)}$$

where ω and ϕ determine the characteristics of the driving force. If the LRA is starting from rest then the initial conditions are given by $x(0)=0$, $x'(0)=0$.

The steady-state maximum amplitude is given by

$$A_{max}(t)=\frac{f}{m\omega\sqrt{(2\omega_0^2\zeta)^2+\frac{1}{\omega^2}(\omega_0^2-\omega^2)^2}} \quad \text{(Eq. 102)}$$

If we were to have n identical LRAs configured in parallel such that all the vibration directions were oriented along e.g. the x-axis, then each LRA would be governed by its own equation

$$x_1''(t) + 2\zeta\omega_0 x_1'(t) + \omega_0^2 x_1(t) = \frac{f_1 \sin(\omega_1 t + \phi_1)}{m} \quad \text{(Eq. 103)}$$

$$x_2''(t) + 2\zeta\omega_0 x_2'(t) + \omega_0^2 x_2(t) = \frac{f_2 \sin(\omega_2 t + \phi_2)}{m}$$

$$\vdots$$

$$x_n''(t) + 2\zeta\omega_0 x_n'(t) + \omega_0^2 x_n(t) = \frac{f_n \sin(\omega_n t + \phi_n)}{m}$$

and the total response of the embedding system, e.g. a phone or game controller, would be proportional to $x(t)=x_1(t)+x_2(t)+\ldots+x_n(t)$. We can reduce the number of equations though by introducing a change of variables. Let

$$X(t)=x_1(t)+x_2(t)+\ldots+x_n(t) \quad \text{(Eq. 104)}$$

which leads to

$$X'(t)=x_1'(t)+x_2'(t)+\ldots+x_n'(t) \quad \text{(Eq. 105)}$$

and

$$X''(t)=x_1''(t)+x_2''(t)+\ldots+x_n''(t). \quad \text{(Eq. 106)}$$

Summing the left-hand sides of Eq. 103 we have

$$(x_1''(t)+x_2''(t)+\ldots+x_n''(t))+2\zeta\omega_0(x_1'(t)+x_2'(t)+\ldots+x_n'(t))+\omega_0^2(x_1(t)+x_2(t)+\ldots+x_n(t))=X''(t)+2\zeta\omega_0 X'(t)+\omega_0^2 X(t) \quad \text{(Eq. 107)}$$

which we recognize as the equation of motion for a single simple harmonic oscillator whose position is given by X(t). On the right-hand side we are left with the sum of the individual forcing functions. Generically there is no simplification of the addition of sine functions. Note though that we can trivially deal with anti-parallel components by reversing the sign of their forcing function. Thus our array of n (anti-)parallel LRAs is governed by the single equation

$$X''(t) + 2\zeta\omega_0 X'(t) + \omega_0^2 X(t) = \left(\frac{f_1 \sin(\omega_1 t + \phi_1)}{m} + \frac{f_2 \sin(\omega_2 t + \phi_2)}{m} + \ldots + \frac{f_n \sin(\omega_n t + \phi_n)}{m}\right) \quad \text{(Eq. 108)}$$

Initial Conditions

Interestingly, the initial condition for the array of LRAs is just the sum of initial conditions of the individual LRAs:

$$X(0)=x_1(0)+x_2(0)+\ldots+x_n(0), X'(0)=x_1'(0)+x_2'(0)+\ldots+x_n'(0), \quad \text{(Eq. 109)}$$

This can have unexpected consequences. Consider the case where there is an even number of LRAs with an identical steady-state maximum amplitude and the steady-state motions of each successive pair are out of phase by π-meaning that when 1, 3 and 5 are at A; 2, 4 and 6 are at −A. Even though each individual element is vibrating, the sum of the amplitudes is always zero and there will be no net motion of the embedding system. If one were to change the driving force while the array is in this state, it will have the same response as if it were starting from rest.

B. Analysis of Multiple LRAs

In the following sections we will use LRAs to give example control effects for each of the six dimensions of SAVANT. Since arrays of LRAs are governed by one differential equation, the analysis of multiple LRAs is computationally no different than that of a single LRA. Physically though, there are advantages. If a single LRA can be driven with a maximum driving amplitude F, then the n-LRAs system is effectively a single LRA with a maximum driving amplitude of n×F. The addition of maximum driving amplitudes forms the basis of the performance, bandwidth and magnitude modes.

1. Magnitude Mode

A straightforward consequence of having multiple LRAs in a synchronous topology is that their response amplitudes (and therefore their momentum transfers to the embedding system) simply add together. In the case of two identical, parallel LRAs driven identically, Eq. 108 yields

$$X''(t) + 2\zeta\omega_0 X'(t) + \omega_0^2 X(t) = 2\frac{F}{m}\sin(\omega t + \phi), \quad \text{(Eq. 110)}$$

which is the equation of a single LRA with a maximum driving amplitude of 2F. Physically this means that if there is a single LRA that can produce a maximum acceleration of 2 g, then two parallel LRAs will produce a maximum acceleration of 4 g; three LRAs could produce 6 g, and so on. This is due to the fact that LRAs are linear systems, as shown in the previous section. This is not the case for rotating actuators such as ERMs, though the methods of SAVANT are still generically applicable to them. The use of multiple actuators to produce vibrational magnitudes greater than the capacity of a single component actuator is denoted herein as a SAVANT running in magnitude mode.

2. Reliability Mode

Having multiple LRAs is in the system allows the designer to build in redundant components. This could be particularly useful especially for medical, scientific or military applications or in situations where the embedding device is habitually used in a rugged environment. When a device contains a group of identical actuators designed for the sole purpose of having built-in redundancy to safeguard against the failure of component actuators, this group of actuators is described as a SAVANT running in reliability mode.

3. Performance Mode

Actuators like LRAs are attractive because they are relatively low-cost, their design and control characteristics are well-understood and because their manufacturing infrastructure is already well-developed. Current trends disfavor the use of these actuators though because of their relatively long response times and narrow bandwidths compared to more exotic actuators-such as piezo-based solutions. By briefly leveraging the increased maximum driving amplitude afforded by an array of LRAs, one can bring the total response time down to the same order of magnitude as the piezo actuators. In the language of the architecture laid out in this work, the use of multiple actuators to effectively emulate a single, high-performance actuator will be referred to as a SAVANT running in performance mode.

To see how a SAVANT in performance mode compares with individual LRAs, the spring characteristics of a single LRA were modeled from data provided by Texas Instruments, Inc. We call this model "MOD1." The data was taken from an AAC ELV1411A LRA from AAC Technologies Holdings Inc. which has a rated resonant frequency of 150 Hz and is shown in FIG. **101**. In this figure the scale of the x-axis is 10.0 ms/div and the y-axis is 100 mV/div.

As shown in this figure, the relative heights and positions of the peaks are determined by the resonant frequency, the driving frequency and the damping ratio. The overall scale for the y-axis is determined by the forcing amplitude and the mass. One can fit the parameters of the spring equation, Eq. 101, by assuming the driving function is at the same frequency as the resonant frequency; i.e. $\omega=\omega_0$. Since we are only interested in relative response times, the resulting motion may be rescaled by the steady-state maximum amplitude given by Eq. 102. This allows one to easily see the fraction of the steady-state maximum amplitude as a function of time.

The damping ratio was derived by fitting a damped, driven harmonic oscillator with resonant frequency of 150 Hz (driven at the resonance frequency) to the experimental output in FIG. **101**. This ratio was found to be $\zeta=0.135$. The resonant LRA frequency is assumed to be 150 Hz and the phase of the forcing function is assumed to be 0 unless otherwise stated. All amplitudes are normalized to the maximum driving amplitude available. Thus an "amplitude of 0.2" refers to an amplitude equal to ⅕ the maximum forcing amplitude. The generic harmonic oscillator equation of motion, Eq. 101 along with these experimentally derived parameters constitute the model MOD1 used extensively throughout this disclosure.

Solving Eq. 101 using the constants for the AAC ELV1411A 150 Hz LRA will give the response of the LRA as a function of time. In FIG. **102** the absolute value of this function is plotted. It is noted in this figure when the LRA reaches 10%, 50% and 90% of its maximum amplitude. The response times for these amplitudes are roughly 1.53 ms, 6.05 ms and 19.75 ms respectively. Although our LRA model, MOD1, of the AAC ELV1411A may be a simple and basic approximation using a generic linear spring LRA model, nevertheless it is useful for our explanation of the SAVANT architecture as follows below.

Now consider the effect of two LRAs working together. The most basic control scheme that can be applied is one where at $t=0$ both LRAs are driven with their maximum forcing amplitudes for some short time τ. Then at $t=\tau$ the forcing amplitudes are reduced for each LRA to half the maximum. By running two parallel LRAs together at half their maximum amplitude the resultant steady-state amplitude will equal that of a single LRA driven at the maximum amplitude. This idea trivially generalizes to n LRAs: we initially drive all LRAs at their maximum forcing amplitudes and then after some time the forcing amplitudes are reduced to 1/n; such that the sum of the forcing amplitudes is equal to the maximum forcing amplitude of a single LRA. It is essential that each component is still driven at 1/n, rather than, e.g., driving one at maximum and keeping the others at rest. The reason for this is that undriven LRAs parallel to the driving force will begin to oscillate and act as dampers on the system.

In FIG. **103** the response of the two-LRA system is presented as a function of time and it is plotted along with the single LRA case. The optimal switching time τ_2 was determined numerically to be $\tau_2=4.36$ ms. For or the two-LRA system, the response times are: 1.16 ms, 2.35 ms and 6.02 ms for 10%, 50% and 90% respectively. One can see the response times are significantly faster, especially the time to reach 90% max amplitude, which is almost 70% faster than the single LRA. Note that the 2-LRA 90% time is roughly the same as the 1-LRA 50% time, as expected. also it is noted that with two LRAs, the system has already reached its maximum amplitude in the second half-cycle.

By adding a third LRA to the system one can achieve even faster response times. In FIG. **104** a 3-LRA system is plotted along with the 2-LRA and 1-LRA systems. Again, the optimal switching time τ_3 was determined numerically to be $\tau_3=2.57$ ms. The 3-LRA system responds the fastest with 10%, 50% and 90% max amplitude times of 1.00 ms, 1.91 ms and 2.66 ms. A 90% max amplitude response time of 2.66 ms is squarely in competition with piezoelectric actuators. The 90% max amplitude response time for the 3-LRA system is over 55% faster than the 2-LRA system and over 86% faster than the single LRA. In Table 101 we summarize the response times for each system. Moreover we see that the 3-LRA system reached 100% of its steady-state maximum amplitude in the first half-cycle—this fact signals that this control sequence is actually the optimal control sequence, if we consider only sinusoidal forcing functions.

TABLE 101

Response Times for Multiple LRAs

Number of LRAs	10% Max Amp. (ms)	50% Max Amp. (ms)	90% Max Amp. (ms)	Increase over 1-LRA
1	1.54	6.05	19.75	0%
2	1.16	2.35	6.02	69.5%
3	1.00	1.91	2.66	86.5%

Response times for 1-LRA, 2-LRA and 3-LRA systems based on our LRA model, MOD1.

4. Bandwidth Mode

Performance mode leverages multiple actuators to emulate a single actuator with an improved response time. The same emulated increase can also be seen in bandwidth. Amplitude response for a typical LRA drops off exponentially as the driving frequency differs from the resonant frequency. The response amplitudes are additive though; meaning that at any given frequency, the maximum amplitude for an n-LRA system is n times the maximum amplitude of the 1-LRA system. We denote systems of multiple, synchronized actuators controlled in a manner as to emulate a single high-bandwidth actuator as a SAVANT running in bandwidth mode.

In the specific case of the our first LRA model, MOD1, when the driving frequency is less than half of the resonant frequency, the steady-state maximum amplitude drops to roughly ¼ of the maximum resonance amplitude. Thus, with four parallel LRAs one can achieve the steady state maximum amplitude even when the driving force frequency differs greatly from the resonant frequency. In FIG. **105** we plot the frequency response curves for 1-, 2-, 3- and 4-LRA systems. The frequency response curves in FIG. **105** from bottom to top are: dotted: 1-LRA; dot-dashed: 2-LRA; dashed: 3-LRA; solid: 4-LRA. We can see from the graph that even extremely low driving frequencies can result in amplitudes greater than the single-LRA resonant-frequency amplitude.

For the system of four 150 Hz LRAs using our LRA model, MOD1, the maximum amplitude while being driven at 1.5 Hz is still 6% higher than a single LRA being driven at its resonant frequency. The response of the 4-LRA system when driven at 1 Hz is almost 90% of the single-LRA amplitude at resonance.

Thus, rather than continually developing more and more exotic actuator solutions, one can use the SAVANT architecture to emulate an LRA with an arbitrarily large bandwidth and force output and/or arbitrarily short response time by adding together an arbitrary number of LRAs.

5. Spatial Mode

So far we have only considered parallel configurations of LRAs. But by orienting groups of LRAs along different axes and synchronizing their control schemes, we can also produce spatial vibrational effects. Examples of such effects are linear vibrations along any line within the space spanned by the vibration axes and circular or elliptical vibrations in any plane within the vibration-axes space. When the system of LRAs is working to produce a spatial vibrational effect, we say that it is running in spatial mode.

Start by considering the action of a single LRA. An LRA is effectively a spring whose mechanical vibrations are oriented along an axis. When the LRA is embedding inside a larger system, it tends to produce vibrations of that embedding system along the axis of orientation. Now add in another LRA: one whose axis of mechanical vibration is oriented perpendicular to the first LRA. Since any two non-collinear lines span the plane, it is not necessary for the two LRAs to be completely orthogonal, only that they not be completely parallel. We choose orthogonal examples for simplicity but more complicated control schemes for non-orthogonal actuators can be devised using the methods of linear algebra.

Each LRA will be vibrating along its own axis, but will each be applying a force to the embedding system. Since the forces they're applying are vectors, we need to take the vector sum to find the resultant force; and therefore the resultant motion of the embedding system. If these two identical, perpendicular LRAs are being driven by identical forcing functions, then the resulting motion of the embedding object will be sinusoidal in the plane spanned by the two axes of orientation. The motion of the embedding object will have an angular separation of 45° from either axis, effectively bisecting the vibrational axes (see FIG. **106**). FIG. **106** shows the resultant motion for orthogonal springs driven with the same amplitude and phase. These four snapshots are taken from the steady state motion of two LRAs. The line represents the vector sum of the LRA displacements.

The angle of the resulting motion can be changed by changing the relative amplitudes of the forcing functions. If we characterize the resulting motion as the angle it makes with the horizontal axis, then any arbitrary linear motion can be achieved with forcing amplitudes proportional to {cos(θ), sin(θ)}, where the first is the forcing amplitude of the horizontal actuator and the second is the forcing amplitude for the vertical actuator. For the example above, we wanted an angle of 45°,

$$\{\cos(45°), \sin(45°)\} = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\},$$

which means that we need to drive the actuators with the same forcing amplitudes. If instead we wanted to make an angle of 135°, i.e. the mirror-flip of the 45° example, we would drive them with amplitudes proportional to

$$\{\cos(135°), \sin(135°)\} = \left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}.$$

This effectively means we would drive them out of phase by 180°. FIG. **107**A shows what the resulting motion would look like.

The direction of the linear motion can even be made to vary in time. In this case, the forcing amplitudes become proportional to {cos(θ(t)), sin(θ(t))}; where now the angle θ is taken to be a function of time. FIG. **106**B shows a linear vibrational effect where the orientation of the line moves from horizontal to vertical. The time variation of θ can be a pre-defined function specified for particular user experiences or the value of θ can be continually or continuously updated by external sensors or user interface. An example of the latter would be a device with a linear vibrational effect that is always oriented north or along the radial line between the device and the center of the Earth. Another example would be a device with a linear vibrational effect capable of tracking an external object, such as a lost pet; or a location, such as a store in a mall.

In FIG. **107**A we have an illustration of two orthogonal springs driven with the same amplitude but out-of-phase by 180°. In FIG. **107**B (we have an illustration of a linear vibrational effect with a time-varying direction. The line represents the vector sum of the LRA displacements. The amplitudes are given by cos(θ(t)) and sin(θ(t)) where θ(t) is taken to be a positive, slowly oscillating function of time, for example:

$$\theta(t) = \frac{1}{2}(\cos(t/250) + 1).$$

Instead of forcing the two orthogonal LRAs with the same phase but different amplitudes, we can also produce spatial vibrational effects by forcing them with the same amplitude but different phases. In the case above, forcing them with the same amplitude but relative phases of 180° again produced a linear vibration. Instead if we forced them with the same amplitude but relative phases of 90° we can produce circular vibrations, as shown in FIG. **108**A. In the case of circular vibrations, the magnitude of the forcing amplitude will determine the radius of the circular vibration. The relative signs of the forcing amplitudes determine the direction of the rotation. In analogy with the linear vibrations of FIG. **107**B, the radius of the circular vibration effect can be made to vary in time by varying the amplitudes. FIG. **108**B shows a circular vibration effect with the radius increasing with time. Again the time variation of the control effect can be pre-scripted or it can be informed by external information.

When the amplitudes of the two vibrational directions are different but they are still forced relatively out-of-phase by 90°, the resulting motion is elliptical, like that shown in FIG. **109**. If we force the two LRAs with different amplitudes and different phases, then we can produce any generic elliptical pattern, as shown in FIG. **110**A. We can also make this generic elliptical pattern change over time by making the phases and amplitudes time-dependent functions as illustrated in **110**B.

The linear and elliptical vibration effects described above are created with proximal SAVANTs: groups of actuators (also known as SAVANT nodes) that are designed to have or appear to have point-like origination. Alternatively, one can create distal SAVANTs; which are which are groups of actuators that are separated physically but still physically attached together so as to create effects originating from multiple points or even extended geometries.

For an example configuration of distal SAVANTs running in spatial mode, consider a bar-like object with uniform mass distribution and two SAVANTs, one placed at either end. Each of the SAVANTs are oriented parallel to each other but perpendicular to the axis-of-symmetry of the object. If the SAVANTs are forced with the same amplitude but out-of-phase by 180°, they will produce a torque on the object, creating a rotational vibrational effect. In principle any arrangements of non-collinear (non-coplanar) LRA orientations will span 2-space (3-space), but maximal orientations will provide the easiest orientations. For proximal systems these will be orthogonal orientations; distal systems are often best implemented by orthogonal, triangular and tetrahedral configurations. Orientations along the edges of higher-vertex Platonic solids (e.g. a cube, an octahedron, etc.) could also produce robust haptic possibilities.

Lissajous and Geometric Transformations

The above examples of the line, circle and ellipse vibration effects are members of the set of Lissajous curves (see FIG. **111**). Various Lissajous curves may be synthesized by a SAVANT having a sufficient design and furthermore that geometric transformations including as examples: rotation, scaling and reflection may be used separately or in various combinations to synthesize a wide gamut of vibration effects. These geometric transformations may take place in a space of higher dimension than the Lissajous curve—for example, a linear vibration (1D) may be rotated in the plane (2D) or 3-space (3D) and a circular vibration (2D) may be rotated in 3-space (3D). Furthermore, a control effect may superimpose multiple Lissajous curves such that, for example, a helical vibration effect may be achieved by having one SAVANT subset generate a circular vibration of frequency ω and a second SAVANT subset generating a linear vibration also of frequency ω that is collinear with the circle's normal vector passing through its center of rotation.

These control effects may have parameters which control the geometric transformations, for example: rotation, scaling and reflection. The parameters may be constant or vary during the effect. The parameters may correspond to real-world events, such as having a consumer electronic device containing a SAVANT, such as a remote control, generate a circular vibration that scales in magnitude based on its proximity to a fixed point in space. Alternatively, a parameter may vary with the distance to a real moving target, or a virtual moving target may only exist as a computer-generated object that is rendered haptically, and which may displayed optionally simultaneously in other sensory modalities for example vision or audition.

In many instances of designing real-world devices, the designer may be required to fit the physical components of a SAVANT into a form factor that does not allow optimum orientations. However, as long as the displacement directions (or axes of rotation for ERM-type SAVANTs) are not collinear then one can still create control effects that span the plane by decomposing the motion vectors into orthogonal components using the methods of linear algebra. An example of this might be a hand-held controller (such as the Xbox 360 Wireless Controller from Microsoft Corporation) that includes two non-collinear ERMs whose design placement maximizes the user's proximity to the actuators. In this case the axes of rotation of the ERMs are not orthogonal nor are they collinear. Thus to create synchronized control effects for these two ERMs one must first decompose the relevant vectors into orthogonal components and then refine the orthogonal actuator control schemes accordingly.

6. Temporal Mode

All of the SAVANT systems explored thus far have been homogeneous systems. We can also consider the possibilities offered by heterogeneous SAVANTs. For example, consider a system of two parallel LRAs: one with a resonant frequency of 175 Hz and one with a resonant frequency of 180 Hz. By driving them separately at their resonant frequencies, the resulting steady state motion will have a beat pattern, as shown in FIG. **112**. In terms of the SAVANT architecture, when multiple actuators are driven together but at different frequencies, we refer to this as running the SAVANT in temporal mode. Running a SAVANT in temporal mode to create asymmetric waveforms allows for devices to create effects such as pulsating or jerky, inhomogeneous (though still periodic) motion.

Generically the superposition of two oscillators oscillating at two different frequencies will result in a beat pattern. If the two frequencies are integer multiples of one another then the oscillators can be considered as part of a Fourier series. Fourier synthesis of multiple actuators driven at various harmonics of a given fundamental enables a device to approximate any (finite and bounded) vibrational output via Fourier decomposition. For example, FIG. **113** shows an approximation of a Sawtooth wave produced by the steady-state motion of five oscillators. The oscillators are each driven in multiples of the fundamental frequency, f_0=25 Hz, and the amplitudes are given by the coefficients of the Fourier series approximation of the linear function f(t)=t. The curve represents the vector sum of the steady-state displacements of five parallel LRAs. The LRAs have resonant frequencies of 25 Hz, 50 Hz, 75 Hz, 100 Hz and 125 Hz and each is forced independently at its resonant frequency. The relative forcing amplitudes are: 2, 1, ⅔, ½, ⅖; and the relative phases are: 0°, 180°, 0°, 180°, 0°. These amplitudes are the coefficients of the first five terms in the Fourier series expansion of the function f(t)=t.

Prior to the introduction of the General Synchronized Vibration approach as described above and the SAVANT architecture, as described here, the capability to generate haptic vibration waveforms with many harmonics has been extremely limited due to the cost and availability of high bandwidth actuators such as Electro-active Polymers. But by creating synchronized systems of multiple lower-cost vibration actuators, one can leverage the increased bandwidth (see Section 3.2.4) to approximate arbitrary vibrational output by: computing the Fourier series of the function matching the desired output; truncating the series at the desired approximation; and then matching each term in the approximation to a SAVANT.

In the example of the Sawtooth wave above, instead of having five actuators—each with different resonant frequencies-one could instead have five SAVANTs. Each SAVANT would correspond to a term in the Fourier series approximation and each would be designed to provide the appropriate amplitude at the corresponding driving frequency. For example, the first term in the series is

$$2\sin\left(\frac{t}{40}\right),$$

where t is measured in milliseconds and the amplitude of 2 means twice the steady state maximum amplitude of the single 150 Hz LRA. Rather than building an LRA specifically designed to give the appropriate amplitude when driven at the frequency, a SAVANT of eight 150 HZ LRAs could provide the necessary output. The eight LRAs would effectively be a SAVANT running in magnitude and bandwidth modes. The remaining terms in the series approximation would each be created by a SAVANT in the same fashion.

The bandwidth gains from just one extra parallel LRA are significant compared to the single LRA. In FIG. **114** we plot the vibrational effect resulting from the sum of three 2-LRA systems based on our LRA model, MOD1. The first 2-LRA system is being driven at a frequency f_0=22.5 Hz, the second and third at $f_1=2f_0$ and $f_2=3f_0$ respectively. The amplitudes for them are (respectively): 1, 1 and 0.4, in units of the 2-LRA maximum driving frequency. The resulting motion is a quick jerk back-and-forth followed by a rest. FIG. **115** shows the same 2-LRA systems being driven at 4.5 Hz, 9 Hz and 13.5 Hz. Even at these low driving frequencies, the maximum amplitude response is comparable to a single LRA being driven at resonance.

III. Example Control Sequences

In Section II above, we developed some of the individual advantages multiple actuator systems have over single actuators. We now describe how those advantages can be made to work together and lay the framework for how to create a SAVANT system that will produce any desired vibrational output. Any possible vibrational effect can be decomposed into the 6-dimensional control space of the SAVANT architecture. Vibrational effects are only limited by the number and relative orientations of actuators in the SAVANT. The relative orientations of the actuators in a SAVANT will dictate the spatial and temporal characteristics while the number of actuators oriented together in a given direction will determine the performance, bandwidth, magnitude and reliability characteristics.

We define a "control effect" as a set of predetermined sequences of control signals for each actuator in the array, such as the example effects in Sections II(B)(1)-(6) above. They can be triggered by a user interface with the device or by internal triggers, such as information from on-board sensors. Running a SAVANT in a single mode constitutes a control effect which makes use of a single dimension of the 6-dimensional control space. It is easy to construct control effects which simultaneously make use of multiple dimensions of the control space. We refer to these as multi-dimensional control effects.

h-Pulse

Control effects leverage the power of multiple actuators to produce desired mechanical responses without the limitations inherent to single actuators. Let us look at an example for the familiar 3-LRA case:

When the optimal response time control sequence outlined in Section II(B)(3) is used in conjunction with an optimal braking strategy, the resultant waveform is a sequence of an integer number of half-wavelengths. This entire control effect is referred to as an "h-pulse." An h-pulse is defined such that the system starts from rest, vibrates at the maximum amplitude for some time (with no ramp-up time) and then immediately stops vibrating. In FIG. **116** we plot the vibrational output for a 3-LRA system exhibiting an h-pulse. The control effect makes use of the optimal response of the performance mode and the optimal braking strategy. The curve represents the summed displacement of three in-phase LRAs. At t=0 ms, apply to each LRA a forcing function with amplitude 1. At t=2.57 ms change the forcing amplitudes to ⅓. At t=51.07 ms change the forcing amplitudes back to 1 and change the phase of each forcing function by 180°. At t=52.33° turn off the forcing functions.

Since the h-pulse control effect must go from maximum mechanical vibration to no vibration, it can have any duration that is an integer multiple of half-wavelengths. For a 150 Hz LRA, since there are two half-wavelengths per cycle, that means one can create h-pulses with durations n×3.33 ms, where n is any integer. This duration can be changed by using pairs of LRAs running in bandwidth mode and driving the SAVANTs at lower frequencies.

Amplitude Seeking

We can arbitrarily rescale an h-pulse by rescaling all of the forcing amplitudes in the control sequence by some scaling factor. This will allow us to have h-pulses of any arbitrary mechanical vibrational amplitude. For instance, in FIG. **117** we have scaled the h-pulse down to 20% of the maximum. Previously, the forcing amplitudes for each LRA were initially 1 (in units of the maximum forcing amplitude); at 2.66 ms the forcing amplitudes were dropped down to ⅓, at 51.07 ms they were shifted out-of-phase by 180° and at 52.33 ms they were set to 0. Now, to create an h-pulse scaled down by 0.2, the forcing amplitudes are initially 0.2×1; at 2.66 ms they change to 0.2×⅓, at 51.07 ms they are shifted out-of-phase by 180° and at 52.33 ms they are set back to 0. The control sequence is identical except that all forcing amplitudes are scaled by a factor of 0.2.

We can use sequences of scaled h-pulses to create an amplitude seeking control scheme, as shown in FIG. **118**. Here the mechanical output of the system is a sequence of half-wavelengths, each of a desired amplitude. In this example we have specified the system to have mechanical amplitudes of 1, ⅕, and ¾ the maximum amplitudes. The scaled h-pulse control sequences were initiated at 0 ms, 10 ms and 20 ms and the system took about 3 ms to reach the desired amplitude. The curve represents the summed displacement of three in-phase LRAs. Each LRA is driven identically and the forcing amplitude can be represented as this piecewise function of time (where t is measured in milliseconds):

$$\mathcal{A}(t)=\begin{cases}1 & 0<t<2.57\\ \frac{1}{3} & 2.57<t<4.24\\ -1 & 4.24<t<5.5\\ 0 & 5.5<t<10\\ \frac{1}{5} & 10<t<12.57\\ \frac{1}{15} & 12.57<t<14.24\\ -\frac{1}{5} & 14.24<t<15.5\\ 0 & 15.5<t<20\\ \frac{3}{4} & 20<t<22.56\\ \frac{1}{4} & 22.56<t<24.234\\ -\frac{3}{4} & 24.24<t<25.5\end{cases}$$

The negative amplitudes can be implemented with positive forcing amplitudes but changes in the forcing phase by 180°. The forcing functions are turned off at t=25.5 ms.

It is possible to create amplitude seeking control sequences with even less time between pulses, but due to the overlap from previous pulses it requires a more detailed algorithm analysis.

Performance ⊗ Magnitude

A simple example of a multi-dimensional control effect is performance ⊗ magnitude (see the discussion below for an explanation of the tensor product notation). The purpose of performance mode is to have a faster response time than a single actuator; the purpose of magnitude mode is to have a stronger mechanical vibration than a single actuator. We can combine these modes when we want to mimic a single actuator that is both faster and stronger than any available component actuator. For the specific case of the 150 Hz LRA discussed above, it was found that the optimal response time required 3 LRAs. Let us then construct a system with 6 LRAs, all oriented along the same axis. We want to position the LRAs as tightly as possible so that the resulting effect will be as point-like as possible. To achieve the performance ⊗ magnitude control effect (assuming the LRAs are initially at rest):

1) Drive each LRA with a sinusoidal forcing function at the maximum amplitude. The phase won't affect the results much.

2) After some pre-determined optimal time τ, change the forcing amplitudes to ⅙ of the maximum amplitude. For the AAC ELV1411A 150 Hz LRA this optimal time is T=2.57 ms.

3) After the desired vibration duration, stop forcing the LRAs. The vibration will damp out in the same time it would take a single LRA to damp out. Alternatively one could apply a braking method.

We note that the control schemes can generically be broken up into two phases: intervals where the SAVANT is outperforming a single component actuator and intervals where the SAVANT is emulating a single component actuator.

Performance ⊗ Magnitude Revisited

We can use performance mode to create extremely fast response and braking times and these allows us to create scaled h-pulses. We can concatenate sequences of these scaled h-pulses to create haptic gradients. These gradients can be scripted effects triggered by sensor information or controlled by a user interface such as a touchscreen. Relying on peoples' inherent ability to detect gradients, these systems could be used as input devices or as devices that give location feedback for the vision impaired. We can make the mechanical output of these gradients arbitrarily large by using multiple 3-LRA systems, effectively leveraging the magnitude mode.

FIG. **119** shows an example of a vibrational gradient produced with a 3-LRA system. The mechanical vibration of the system is ramped up in discrete steps in response to a user input. The curve represents the summed displacement of three in-phase LRAs. Each LRA is driven identically and the forcing amplitude for each is represented as this piecewise function of time (where t is measured in milliseconds):

$$\mathcal{A}(t) = \begin{cases} \frac{1}{15} & 2.57 < t < 49.24 \\ \frac{2}{15} & 53.99 < t < 100.66 \\ \frac{1}{5} & 0 < t < 2.57 \vee 104.82 < t < 151.49 \\ \frac{4}{15} & 157.52 < t < 204.19 \\ \frac{1}{3} & 210.85 < t < 257.53 \\ \frac{2}{5} & 51.42 < t < 53.99 \\ \frac{3}{5} & 102.25 < t < 104.82 \\ \frac{4}{5} & 154.95 < t < 157.52 \\ 1 & 208.29 < t < 210.85 \end{cases}$$

The forcing functions are turned off at t=257.53 ms.

Bandwidth ⊗ Performance ⊗ Magnitude

The widths of the single-peak h-pulses described in the Amplitude Seeking subsection were set by the driving frequency. Those h-pulses were created by a SAVANT composed of three parallel LRAs. By adding another set of three parallel LRAs we can increase the vibrational magnitude and bandwidth of the SAVANTs. This will allow the SAVANTs to be driven at lower frequencies while still maintaining on-resonance amplitudes. In FIG. **120** we show the motion of a system composed of two 3-LRA systems. The control schemes for both 3-LRA systems are identical and the schemes are very similar to that described for the single-peak h-pulses; only the amplitude switching times have been changed. The major difference though is that in this case the LRAs are being driven at 25 Hz instead of the resonant frequency of 150 Hz.

FIG. **120** illustrates an Elongated h-Pulse created by 150 Hz LRAs using our LRA model, MOD1, driven at 25 Hz. The redundancy of LRAs allows the vibration amplitude to remain comparable to a single one of our LRA models, MOD1, driven at its resonance. Note that the force output is not comparable though because the force scales as the square of the oscillation frequency.

Even though the MOD1 LRAs in this group are being driven at $\frac{1}{6}^{th}$ of their resonance frequency, their total vibration amplitude is still greater than a single LRA based on our LRA model, MOD1, vibrating at resonance. Note that the total force output of this group of LRAs based on our LRA model, MOD1, is not comparable to that of a single model LRA vibrating at resonance because the force output scales with the square of the vibration frequency. This control scheme is an example of SAVANTs utilizing bandwidth, magnitude and performance modes simultaneously. From the example multi-dimensional control schemes above that, given enough actuators, it is possible to simultaneously control a SAVANT in any subset of the six modes of the SAVANT architecture.

As mentioned above, a SAVANT may incorporate a wide range of vibration actuators including a linear resonant actuator which has a moving mass that oscillates back and forth along a linear path. Other actuators that are useful in a SAVANT include the actuators where the mass is constrained to rotate about a particular vibration axis, following a circular or semi-circular path. These rotational actuators include Eccentric Rotating Mass actuators, Pivoting Mass actuators, and Rocking Mass Actuators as have been introduced earlier in this disclosure. While a single linear resonant actuator can impart a force on the mounting plate it is attached to, a single rotational actuator can impart a torque onto the mounting plate it is attached to. As explained previously in this disclosure, the torques generated by one or more rotating vibration actuators when added with torques generated by one or more counter-rotating vibrating actuators can sum to a net force upon a mounting plate, for example in cases when the axes of rotation of the rotational actuators are arranged collinearly.

Current controllers, such as Nintendo's Wii Remote Plus motion controller and Sony Computer Entertainment's PlayStation Move motion controller, could be augmented at little cost to include a SAVANT that provides additional haptic cues and vibrations. These could be in addition to or instead of the ERMs currently in the devices. Though the incremental cost of adding controllers is small, the potential revenue from the expanded possibility of titles could be very high. Moreover, as other companies move towards controller-less motion controlled gaming and computing, haptic feedback provides an advantage for hand-held controllers. At the same time, it is possible though to develop wearable haptic devices, such as wristbands or belts incorporating SAVANTs, which provide spatial and directional feedback. These devices could be in addition to or in lieu of a hand-held controller.

Other wearable applications include a SAVANT incorporated into a vibration device contained within a wristband, an armband or a leg band. Still other examples of wearable applications include a SAVANT incorporated into a vibration device contained within wearable accessories such as a pair of eyeglasses, a pair of headphones or a hearing aid. Still further applications could include incorporating a SAVANT into a vibration device that is used for providing haptic feedback in a handheld stylus drawing or writing or pointing tasks.

Furthermore, SAVANTs can be incorporated into personal pleasure devices to increase the range of haptic vibration sensations for a person's body, for usage of such haptic vibration devices either externally or internally in relation to a person's body, or both. A SAVANT may be used for other hand tools and devices that are commonly used in construction; for example screwdrivers, hand-drills; pliers; and wrenches.

Another application might be a SAVANT in a device that augments reality for a non-sensory impaired person, or alternatively in a device that substitutes vibratory feedback and sensations for sensory modalities for which a person may have a deficit such as vision or hearing.

SAVANTs may be used to telepresence activities, such that a parent who is located in one city is able to hold a SAVANT enabled device, such as a game controller, that produces vibrations that correspond and convey, for example, to the respiration of the parent's child located in another city, or the heartbeat of a parent conveyed to the parent's child to enable a parent-child connection. A vibration device incorporating a SAVANT may be used to help generate calming vibration patterns for a person having Autism Spectrum Disorder. SAVANTs may also be used to generate vibrations in children's toys, for example inside plush toys, such as a teddy bear.

The SAVANT architecture is intentionally designed as a scalable network topology such that a first vibration device with a first embedded SAVANT may physically and logically interface with a second vibration device with a second embedded SAVANT such that the vibration controllers of both vibration devices could share sensors, data and control to establish a larger SAVANT device. In this fashion, the SAVANTs embedded inside various devices may be considered as modular vibration units that can be attached together to form larger SAVANTs with different and often more robust capabilities.

IV. Additional

A. Discussion of Optimal Control Methods

It is clear from FIGS. **102-104** that additional sinusoidal forcing terms can change the relative heights of the response peaks, but they can't change the temporal distribution, regardless of the switching time τ. This is because the forcing functions applied to each LRA all have the same frequency and phase. Thus before $t=\tau$,

$$\left(\frac{f_1\sin(\omega_1 t+\phi_1)}{m}+\frac{f_2\sin(\omega_2 t+\phi_2)}{m}+\ldots+\frac{f_n\sin(\omega_n t+\phi_n)}{m}\right)=\left(\frac{f_1}{m}+\frac{f_2}{m}+\ldots+\frac{f_n}{m}\right)\sin(\omega_0 t). \quad \text{(Eq. 111)}$$

And after the switching time τ, the left-hand side of Eq. 111 is merely rescaled by a factor of 1/n.

(Note that for all of the analysis in Section 3.2.3, we've only considered sinusoidal forcing functions with the phase $\phi=0$. This phase will give you the fastest 3-LRA response because it allows the system to reach the maximum amplitude in the first half-cycle. For a 2-LRA system though, driving functions with a phase $\phi=\pi/2$ give a slightly faster 90% max amplitude response time of roughly 4.8 ms.)

It is well known from the theory of linear differential equations that a sinusoidally-driven, damped harmonic oscillator has a response function whose frequency and phase are determined solely by the oscillator characteristics and the frequency and phase of the driving function. Additional LRAs only change the amplitude of the driving function (Eq. 111) and we're only considering identically LRAs in this particular analysis. Therefore, it is impossible to change the position of the first peak by adding or removing LRAs from the system. Hence, since the 3-LRA system has its first peak at maximum amplitude, the control method we propose here is the optimal control method, within the subset of control methods where each LRA is driven by a sinusoidal function with frequency $\omega=\omega_0$.

This analysis assumes that the maximum allowable driving amplitude is the steady-state maximum amplitude. Many actuators can be "over-driven" though; such that they can be driven beyond their steady-state maximum amplitude for a brief period of time. If this is a possibility then the 3-LRA optimal control scheme is equivalent to a 2-LRA control scheme where they are initially driven at 1.5×steady-state maximum amplitude or a single LRA that is briefly driven at 3×steady-state maximum amplitude.

1. Discussion of the Frequency and Amplitude Response Functions

When a harmonic oscillator-type vibration actuator is driven by a constant sinusoidal external force there are two functions that characterize the response of the vibration actuator. The first is the amplitude response, defined as how the amplitude of the resulting motion increases or decreases with time in the presence of an external driving force. For the previously discussed LRA model, MOD1, the vibration actuator reaches 10%, 50% and 90% of its maximum amplitude in roughly 1.53 ms, 6.05 ms and 19.75 ms respectively. Alternatively one could say that these happen in the first, second and sixth half-wavelength. It is desirable to achieve a faster amplitude response.

The second important relationship is how the amplitude responds to the driving force frequency. This relationship is called the bandwidth of the vibration actuator. We shall denote this as the "frequency response". Typically the amplitude responds maximally to the resonant frequency and the response diminishes as the driving force frequency diverges from the resonant frequency. Using the same 150 Hz LRA as mentioned above, the maximum amplitude response happens for a sinusoidal driving force with a frequency of 150 Hz, while a driving force with frequency 105 Hz only produces an amplitude of roughly half the maximum.

Each response function mentioned above is the result of inherent physical limitations associated with harmonic oscillators. These natural limitations of the vibration actuators can be mitigated by employing multiple vibration actuators in parallel. Without loss of generality we shall consider only identical vibration actuators though all that is discussed applies to a heterogeneous population of vibration actuators as well.

For example, when driven at resonance our model of an LRA, MOD1, reaches about 33% of its maximum amplitude in the first quarter-wavelength. Leveraging this fact, one can drive of our MOD1 model LRAs together at resonance and after one quarter-wavelength the summed response amplitude of the three LRAs will roughly be the same the maximum amplitude of any single MOD1 model LRA. Thus we say that these three LRAs can emulate a single LRA that has the maximum amplitude response in minimal time (that is to say, a quarter-wavelength, since for a harmonic oscillator starting from rest and driven by a sinusoidal force, the first quarter-wavelength corresponds to the first inflection point and therefore the first extremum of the motion).

The same approach can be applied to improving the frequency responses to overcome bandwidth limitations of any individual vibration actuator. Since a single MOD1 LRA responds to a 105 Hz driving force with roughly half its maximum possible amplitude, by driving two MOD1 LRAs together one can emulate the single LRA on-resonance amplitude even when driven at 105 Hz.

When one employs a collection of vibration actuators together to emulate a virtual vibration actuator possessing amplitude and/or frequency responses superior to any component vibration actuator in the collection, we denote this as running a SAVANT in performance and/or bandwidth mode respectively. A wide gamut of vibration output patterns can be built from amalgamations and concatenations of performance and bandwidth modes. These output patterns include, but are not limited to: immediate cessation of vibrations; discrete gradients of constant amplitude; and single, unidirectional h-pulses. Various scenarios and examples of control strategies are discussed below.

1) Combining multiple vibration actuators to emulate a vibration actuator with an amplitude response faster than any component vibration actuator. When the virtual vibration actuator goes from zero amplitude to its maximum amplitude within one quarter-wavelength. We denote this vibrational output as optimal amplitude response and the corresponding control strategy as the SAVANT OAR Control Strategy. When the virtual vibration actuator goes from its maximum amplitude to zero amplitude within one quarter-wavelength. We denote this vibrational output as optimal braking and the corresponding control strategy as the SAVANT OB Control Strategy.

2) Combining multiple vibration actuators to emulate a vibration actuator with a frequency response greater than any component vibration actuator, for any given sinusoidal driving force frequency.

3) Combining multiple vibration actuators to emulate a virtual vibration actuator that undergoes optimal amplitude response immediately followed by optimal damping. This effectively mimics a single vibration actuator that exhibits one half-wavelength of sinusoidal motion. We denote this vibrational output as an h-pulse and the corresponding control strategy as the SAVANT HP Control Strategy.

4) Combining multiple vibration actuators to emulate a virtual vibration actuator that exhibits successive h-pulses of arbitrary amplitudes. We denote this vibrational output as amplitude seeking and the corresponding control strategy as the SAVANT AS Control Strategy.

5) Combining multiple vibration actuators to emulate a virtual vibration actuator that undergoes optimal amplitude response followed by constant sinusoidal motion for N>1 half-wavelengths followed by optimal damping. This effectively mimics a single vibration actuator that exhibits N half-wavelength of sinusoidal motion with no periods of reduced amplitude at the onset or offset. We denote this vibrational output as an h-pulse and the corresponding control strategy as the SAVANT IMP Control Strategy.

6) Combining multiple vibration actuators to emulate a virtual vibration actuator that generates a sequence of h-pulses such that successive h-pulses differ in amplitude. The changes in amplitude may or may not be associated with information external to the vibration actuators, such as dynamic position or orientation information. We denote this vibrational output as a vibrational gradient and the corresponding control strategy as the SAVANT VG Control Strategy.

7) Combining multiple vibration actuators to emulate a virtual vibration actuator that exhibits a half-wavelength oscillation of arbitrary width. We denote this vibrational output as an elongated h-pulse and the corresponding control strategy as the SAVANT EHP Control Strategy.

8) Combining multiple vibration actuators to emulate a virtual vibration actuator that exhibits a superposition or sequential concatenation of any one or more of the above mentioned control sequences. This would include for example a control sequence consisting of a sequence of h-pulses with monotonically increasing amplitudes followed by an h-pulse of a given amplitude and duration followed by a sequence of h-pulses with monotonically decreasing amplitudes. This vibrational output is a template for haptically emulating the tactile feedback of a mechanical button press.

2. Alternative LRA-Type Actuators

In the above analysis only a fitted model from data of the AAC ELV1411A 150 Hz LRA has been considered. There are many types of LRAs on the market today though, so it is instructive to test the robustness of the SAVANT architecture for different actuators. One can take same data from FIG. **101** but fit it to a hypothetical LRA with a resonant frequency of 175 Hz. We can then repeat the analysis for this hypothetical LRA; FIG. **121** and Table 102 are summaries of the results. Surprisingly we see that the relative gains are exactly the same as those for the 150 Hz LRA. Each of the three curves corresponds to a 1-LRA, 2-LRA or 3-LRA system. All LRAs are driven in-phase at 175 Hz. For the single LRA the forcing function is set to maximum at t=0 ms. For the 2-LRA system, at t=0 ms, each LRA is driven at maximum amplitude; at t=4.36 ms the driving amplitudes are set to ½. For the 3-LRA system, each LRA is initially driven at maximum at t=0 ms; and at t=2.18 ms the forcing amplitudes are set to ⅓.

TABLE 102

Response Times for Multiple LRAs-175 Hz

Number of LRAs	10% Max Amp. (ms)	50% Max Amp. (ms)	90% Max Amp. (ms)	Increase over 1-LRA
1	1.33	5.19	16.93	0%
2	1.01	2.02	5.16	69.5%
3	0.87	1.64	2.29	86.5%

Response times for 1-LRA, 2-LRA and 3-LRA systems based on fitted model of a 175 Hz LRA.

3. Optimal Braking Methods

For a given control effect, it is possible to predetermine the optimal braking strategy such that the effect can be halted as quickly as possible. This will usually involve using a forcing (braking) function that is 180° out-of-phase with respect to the initial forcing function. For the familiar case of the our model LRA, MOD1, the optimal braking method to go from maximum mechanical vibration to effectively zero mechanical vibration is shown in FIG. **122**. Each LRA in the system is initially forced with an amplitude of ⅓, mimicking the output and response time of a single LRA. A signal is sent at 50 ms to initiate the braking control sequence, as indicated by the second dashed line. At 51.07 ms the forcing functions are shifted out-of-phase by 180° from the original forcing functions, effectively forcing the LRAs with negative amplitudes. At 52.33 ms the forcing amplitudes were set to 0, as indicated by the third dashed line. Because the mechanical output goes from maximum to effectively zero in a quarter cycle, for the parallel 3-LRA system there is no control sequence that will brake the system faster.

FIG. **122** illustrates Optimal Braking of a 3-LRA System. In this case, the braking strategy effectively fully damps the mechanical vibration within a quarter-cycle. The curve represents the summed displacement of 3 LRAs driven in-phase. At t=0 ms each LRA is driven at ⅓. At t=51.07 ms each driving amplitude is set to 1 and the phases are changed by 180°. At t=52.32 ms the forcing functions are turned off.

4. Example Operational Process

An example process according to the above is presented by the following steps

Step 1) Select a desired output waveform.

Step 2) Define a collection of harmonic oscillators to emulate the desired waveform.

Step 3) Set up the equation of motion for the collection of harmonic oscillators, with each driven by its own input function. For any set of identical oscillators this reduces to a single equation driven by the sum of the relevant component input functions. Each discontinuous change in the desired output waveform will generically correspond to a discontinuous change in the input functions. For harmonic oscillators with resonant frequency, f_0, the times at which these changes in the input function should happen is generically of the order

$$\frac{1}{2\pi f_0}$$

before each discontinuity.

Step 4) Solve the equation of motion with a defined set of initial conditions. For a collection of n oscillators trying to emulate a single oscillator with maximum amplitude A, typical initial conditions for various desired effects are: +m A for immediate maximum amplitude; −m A for immediate minimum amplitude (stopping or braking); +A for steady state A; 0 for no amplitude. The necessary number of oscillators needed in order to immediately reach the emulated maximum amplitude within half a wavelength is defined by the characteristic amplitude response function for the oscillator. For our model LRA, MOD1, this number is 3 because in half of a wavelength that particular LRA model has reached about 33% of its maximum amplitude.

Step 5) Slowly vary the amplitudes and/or discontinuity times in the input function until the solution matches the desired output waveform to a sufficient degree of accuracy. Computational tools such as Java applets or Mathematica DynamicModules from Wolfram Research can facilitate this process by enabling smooth manipulation of the input function parameters, as shown in FIGS. **97**-**100**. (Note that the resonant frequency for this oscillator is 150 Hz, which gives

$$\frac{1}{2\pi f_0} = 0.00106103\text{s.})$$

5. Tensor Product Notation

The "circle times" symbol used in Section 4, ⊗, is a symbol used to indicate the tensor product of vector spaces. We are using this notation in a similar fashion to how it is used in quantum mechanics. In quantum mechanics the tensor product is used to describe the resulting multi-particle states available to two or more interacting or entangled particles. If one particle is in state a and another is in state b, the two-particle state would be labeled by a ⊗ b. It represents all possible multi-particle states available given the two particles that make up the state. In the case of SAVANT, each SAVANT is like a quantum particle with available states and the tensor product of those SAVANTs defines the vector space of control effects available to the system.

For example, consider a collection of six parallel LRAs. We can choose to group these LRAs into two groups of three. Each group of three will be referred to as a SAVANT. We can individually (and independently) run each SAVANT in performance mode to emulate a single LRA with enhanced response time. If we choose to synchronize the control of each SAVANT though, such that they are both driven in performance mode at the same time and with the same phase, we would effectively be running the two SAVANTs in magnitude mode. To denote the fact that the individual LRAs in simultaneously in both the performance "state" and the magnitude "state", we would say that they are being driven in performance ⊗ magnitude.

The notation is used for convenience and is not intended to be completely analogous to its usage in tensor algebra.

Various reference materials are mentioned below, and are wholly incorporated by reference here. The reference materials include the following. The white paper from Immersion Corporation, entitled "Haptics in Touchscreen Hand-Held Devices," dated April 2012. This white paper describes in Section 3.1 four types of actuators: Eccentric Rotating Mass Actuators (ERMs), Linear Resonant Actuators (LRAs), Piezo Modules, and Electro-Active Polymer Actuators (EAPs). The SAVANT architecture can be used with all of these types of actuators, and various instantiations of each. The monograph, "Engineering Haptic Devices: A Beginner's guide for Engineers," Thorsten A. Kern, editor, published by Springer-Verlag, copyright 2009.

The monograph, "Vibrations and Waves," by A. P. French, published by W. W. Norton & Company; 1 edition (1971).

The monograph, "Human Haptic Perception: Basics and Applications," edited by Martin Grunwald, published by Birkhauser Verlag; (2008).

The monograph, "Feedback and Control for Everyone" by Pedro Albertos and Iven Mareels, published by Springer-Verlag; (2010).

The blogpost, "Enabling high-definition haptics: introducing piezo actuators," by Eric Siegel of Texas Instruments.

The data sheet, "DRV 8601: Haptic Driver for DC Motors (ERMs) and Linear Vibrators (LRAs) with Ultra-Fast Turn-On," from Texas Instruments Incorporated.

The World Wide Web article "How To Disassemble an Xbox 360 Wireless Controller" from instructables.com.

The following application notes from Precision Microdrives Limited: "AB-002: Discrete H-bridge For Enhanced Vibration Control" and "AB-003: Driving Linear Resonance Vibration Actuators".

Furthermore, the reference by Eric W. Weisstein entitled "Lissajous Curve," from MathWorld—A Wolfram Web Resource.

If there is a conflict or inconsistency between material in the instant specification and any discussion in the aforementioned references, then the specification dominates. This includes any conflicts or inconsistencies with regard to definitions, concepts, jargon, use of language, terminology or the like.

Although aspects of the disclosure have been described with reference to particular embodiments, it is to be understood that these embodiments are merely illustrative of the principles and applications of the present disclosure. It is therefore to be understood that numerous modifications may be made to the illustrative embodiments and that other arrangements may be devised without departing from the spirit and scope of the present disclosure as defined by the appended claims. By way of example only, it is possible to vary aspects of the embodiments herein to some degree while achieving the advantages of the Synchronized Array of Vibration Actuators in a Network Topology architecture and other benefits of the disclosure.

INDUSTRIAL APPLICABILITY

The present invention enjoys wide industrial applicability including, but not limited to, handheld or wearable devices providing haptic sensations or feedback to a user.

The invention claimed is:

1. A vibration device, comprising:

a mounting platform;

a plurality of linear resonant actuators attached to the mounting platform, each of the plurality of linear resonant actuators having a moveable mass and an axis of vibration in accordance with a direction of movement of the moveable mass, the axes of vibration of the plurality of actuators being arranged in a substantially parallel configuration; and

a controller coupled to each of the plurality of actuators, the controller being configured to (1) control each actuator to impart a sinusoidal vibration force of a frequency, f_1, onto the mounting platform, and (2) control amplitudes and phases of the sinusoidal vibration force from each actuator to generate a combined vibration waveform onto the mounting platform;

the combined vibration waveform emulating a virtual vibration actuator having one or both of the following properties: a faster amplitude response than any one of the plurality of linear resonant actuators, or a larger amplitude response than any one of the plurality of linear resonant actuators for non-resonant frequencies.

2. The vibration device of claim **1**, wherein all of the plurality of linear resonant actuators are substantially identical.

3. The vibration device of claim **2**, wherein the vibration device is configured for haptic applications.

4. The vibration device of claim **3**, wherein the vibration device is further configured to be portable by a person.

5. The vibration device of claim **4**, wherein the vibration device is further configured to be worn or held by a person.

6. The vibration device of claim **1**, wherein the vibration device is further configured to provide haptic output for one of the following devices: a game controller, a motion game controller, a handheld game console, a remote control, a handheld portable computer, a navigation device, a handheld construction tool, a handheld surgical tool, a stylus, a plush toy, a pair of eyeglasses, a wristband, a wristwatch, a belt, an armband, a leg band, a mobile phone, a tablet computer, a device for aiding a vision-impaired person, a device for aiding a hearing-impaired person, and a device for augmenting reality with haptic feedback, a personal pleasure device for providing pleasurable haptic sensations, and a vibration device used for singly or in a pair for conveying telepresence.

7. The vibration device of claim **1**, wherein the combined waveform has the larger amplitude response than any one of the linear resonant actuators for non-resonant frequencies, and the virtual vibration actuator has a control sequence that controls the combined waveform to go from zero amplitude to maximum amplitude within one quarter-wavelength.

8. The vibration device of claim **1**, wherein the virtual vibration actuator has a larger amplitude response than any one of the plurality of linear resonant actuators for any given sinusoidal driving force frequency not equal to the resonant frequency of any one of the plurality of linear resonant actuators.

9. The vibration device of claim **1**, wherein the controller manages the virtual vibration actuator to implement a control sequence that undergoes optimal amplitude response immediately followed by optimal damping.

10. The vibration device of claim **1**, wherein the controller manages the virtual vibration actuator to implement a control sequence that exhibits successive h-pulses of arbitrary amplitudes.

11. The vibration device of claim **1**, wherein the virtual vibration actuator is controlled by the controller to implement a control sequence that undergoes optimal amplitude response followed by constant sinusoidal motion for $N>1$ half-wavelengths followed by optimal damping, wherein N is an integer value.

12. The vibration device of claim **1**, wherein the virtual vibration actuator is controlled by the controller to implement a control sequence that generates a sequence of h-pulses such that successive h-pulses differ in amplitude.

13. The vibration device of claim **12**, wherein changes in amplitude have a relationship to at least one of changes in position of the vibration device and changes in orientation of the vibration device.

14. The vibration device of claim **1**, wherein the virtual vibration actuator is controlled by the controller to implement a control sequence that exhibits an h-pulse of arbitrary width.

15. The vibration device of claim **14**, wherein the arbitrary width is determined by a resonant frequency associated with one or more of the plurality of linear resonant actuators.

16. The vibration device of claim **7**, wherein the virtual vibration actuator is controlled by the controller to exhibit superposition or sequential concatenation of multiple instances of the control sequence.

17. A vibration device, comprising:

a mounting platform;

a plurality of vibration actuators attached to the mounting platform, each of the plurality of vibration actuators having a moveable mass that is constrained to rotate about an axis of vibration and a spring attached to the moveable mass and the mounting platform, wherein each of the vibration actuators is configured to operate as a mechanical harmonic oscillator, and wherein the axes of vibration of the plurality of actuators are arranged in a substantially parallel configuration; and

a controller coupled to each of the plurality of actuators, the controller being configured to (1) control each actuator to impart a sinusoidal vibration torque of a frequency, f_1, onto the mounting platform, and (2) control amplitudes and phases of the sinusoidal vibration torque from each actuator to generate a combined vibration torque onto the mounting platform;

the combined vibration torque emulating a single virtual vibration actuator having one or both of the following properties: a faster amplitude response than any one of the plurality of actuators, or a larger amplitude response than any one of the plurality of actuators for non-resonant frequencies.

18. The vibration device of claim **17**, wherein the plurality of vibration actuators is a first plurality of actuators, and the vibration device further comprises:

a second plurality of vibration actuators attached to the mounting platform, each of the second plurality of vibration actuators having a moveable mass that is constrained to rotate about an axis of vibration and a spring attached to the moveable mass and the mounting platform, wherein each of the second vibration actuators is configured to operate as a mechanical harmonic oscillator, and wherein the axes of vibration of the second plurality of actuators are arranged in a substantially parallel configuration with the first plurality of vibration actuators;

wherein the controller is coupled to each of the second plurality of actuators, the controller being configured to (1) control each of the second plurality of actuators to impart the sinusoidal vibration torque of the frequency, f_1, onto the mounting platform such that movement of the moveable masses in the second plurality of vibration actuators counter-rotate relative to movement of the moveable masses in the first plurality of vibration actuators, and the controller is configured to (2) control amplitudes and phases of the sinusoidal vibration torque from each actuator of the second plurality to generate a combined vibration force onto the mounting platform that emulates a single virtual vibration actuator.

19. The vibration device of claim **17**, wherein all of the vibration actuators are substantially identical.

20. The vibration device of claim **19**, wherein the vibration device is configured for haptic applications.

21. The vibration device of claim **20**, wherein the vibration device is further configured to be portable by a person.

22. The vibration device of claim **21**, wherein the vibration device is further configured to be worn or held by a person.

23. A vibration device, comprising:

a mounting platform;

a plurality of linear resonant actuators attached to the mounting platform such that the axes of vibration for the actuators are substantially in a parallel configuration, each linear resonant actuator including a moveable mass; and

a controller coupled to each of the linear resonant actuators, the controller being configured to (1) control each one of the linear resonant actuators to impart a sinusoidal vibration force of a frequency, f_1, onto the mounting platform, and to (2) control amplitudes and phases of each sinusoidal vibration force to generate a combined vibration waveform onto the mounting platform, the combined vibration waveform emulating a virtual actuator having a maximum amplitude, A_1; and

the virtual vibration actuator having one or both of the following properties: a faster amplitude response than any one of the plurality of linear resonant actuators, or a larger amplitude response than any one of the plurality of linear resonant actuators for non-resonant frequencies.

24. The vibration device of claim **23**, wherein the controller is configured to control the combined vibration waveform so that the virtual vibration actuator starts from rest and extends to the maximum amplitude A_1 in one quarter-wavelength of motion with no periods of reduced amplitude oscillation prior to maximal extension.

25. The vibration device of claim **23**, wherein the controller is configured to control the combined vibration waveform so that the virtual vibration actuator transitions from extension to the amplitude A_1 to a rest position in one quarter-wavelength of motion with no periods of reduced amplitude oscillation posterior to maximal extension.

26. The vibration device of claim **23**, wherein the controller is configured to control the combined vibration waveform so that the virtual vibration actuator starts from rest, extends to a desired amplitude in one quarter-wavelength of motion and then returns to rest in the next successive quarter-wavelength of motion, with no periods of reduced amplitude oscillation at the onset or offset of the device's motion.

27. The vibration device of claim **26**, wherein the virtual vibration actuator is controlled by the controller to start from rest, extend to a selected amplitude in one quarter-wavelength of motion, oscillate about its rest position at the selected amplitude for an integer number of half-wavelengths, and then return to rest in one quarter-wavelength of motion, with no periods of reduced amplitude oscillation at the onset or offset of the vibration device's motion.

28. The vibration device of claim **26**, wherein the controller is configured to generate a successive series of the combined vibration waveforms, wherein each one of the waveforms in the successive series has the same amplitude.

29. The vibration device of claim **26**, wherein the controller is configured to generate a successive series of the combined vibration waveforms, wherein each one of the waveforms in the successive series has a different amplitude.

30. The vibration device of claim **23**, wherein the virtual vibration actuator has an oscillation period, T_1, which starts from rest, extends to a selected amplitude in one quarter-period of motion, and then returns to rest in the next successive quarter-period of motion, with no oscillations of reduced amplitude at the onset or offset of the vibration device's motion.

31. The vibration device of claim **23**, wherein one or both of input forcing functions and output vibration patterns managed by the controller are related to either (1) an absolute spatial position and orientation of the vibration device, or (2) a spatial position and orientation of the vibration device relative to an external object, living entity or location.

32. The vibration device of claim **31**, wherein one or both of input forcing functions and output vibration patterns managed by the controller are dynamically determined by the spatial position and orientation of the vibration device relative to an external object, so that the vibration device is configured to guide a user towards or away from the external object, living entity or location.

* * * * *